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# Not So Demanding: Demand Structure and Firm Behavior†

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*We show that any well-behaved demand function can be represented by its "demand manifold," a smooth curve that relates the elasticity and convexity of demand. This manifold is a sufficient statistic for many comparative statics questions; leads naturally to characterizations of new families of demand functions that nest most of those used in applied economics; and connects assumptions about demand structure with firm behavior and economic performance. In particular, the demand manifold leads to new insights about industry adjustment with heterogeneous firms, and can be empirically estimated to provide a quantitative framework for measuring the effects of globalization.* (*JEL* F12, L11)

Assumptions about the structure of preferences and demand matter enormously for comparative statics in trade, industrial organization, and many other applied fields. Examples from international trade include competition effects (such as whether globalization reduces firms' markups), which depend on whether the elasticity of demand falls with sales;[1] and selection effects (such as whether more productive firms select into FDI rather than exports), which depend on whether the elasticity and convexity of demand sum to more than three.[2] Examples from industrial organization include pass-through (do firms pass on cost increases by more than dollar-for-dollar?), which depends on whether the demand function is log-convex;[3] and the welfare effects of third-degree price discrimination, which depend on how

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[1] See Krugman (1979) and Zhelobodko et al. (2012).

[2] See Helpman, Melitz, and Yeaple (2004) and Mrázová and Neary (forthcoming).

[3] See Bulow and Pfleiderer (1983) and Weyl and Fabinger (2013).

demand convexity varies with price.[4] In all these cases, the answer to an important real-world question hinges on a feature of demand that seems at best arbitrary and in some cases esoteric. All bar specialists may have difficulty remembering these results, far less explicating them and relating them to each other.

There is an apparent paradox here. These applied questions are all supply-side puzzles: they concern the behavior of firms or the performance of industries. Why then should the answers to them hinge on the shape of demand functions, and in many cases on their second or even third derivatives? However, as is well known, the paradox is only apparent. In perfectly competitive models, shifts in supply curves lead to movements along the demand curve, and so their effects hinge on the slope or elasticity of demand. When firms are monopolists or monopolistic competitors, as in this paper, they do not have a supply function as such; instead, exogenous supply-side shocks or differences between firms lead to more subtle differences in behavior, whose implications depend on the curvature as well as the slope of the demand function.

Different authors and even different subfields have adopted a variety of approaches to these issues. Weyl and Fabinger (2013) show that many results can be understood by taking the degree of pass-through of costs to prices as a unifying principle. Macroeconomists frequently work with the "superelasticity" of demand, due to Kimball (1995), to model more realistic patterns of price adjustment than allowed by CES preferences. In our previous work (Mrázová and Neary forthcoming), we showed that, since monopoly firms adjust along their marginal revenue curve rather than the demand curve, the elasticity of marginal revenue itself pins down some results. Each of these approaches focuses on a single demand measure that is a sufficient statistic for particular results. This paper goes much further than these, by developing a general framework that provides a new perspective on how assumptions about the functional form of demand determine conclusions about comparative statics.

The key idea we explore is the value of taking a "firm's-eye view" of demand functions. To understand a monopoly firm's responses to infinitesimal shocks it is enough to focus on the local properties of the demand function it faces, since these determine its choice of output: the slope of demand determines the firm's level of marginal revenue, which it wishes to equate to marginal cost, while the curvature of demand determines the slope of marginal revenue, which must be negative if the second-order condition for profit maximization is to be met. Measuring slope and curvature in unit-free ways leads us to focus on the elasticity and convexity of demand, following Seade (1980), and we show that for any well-behaved demand function these two parameters are related to each other. We call the implied relationship the "demand manifold," and show that it is a sufficient statistic linking the functional form of demand to many comparative statics properties. It thus allows us to develop new comparative statics results and illustrate existing ones in a simple and compact way; and it leads naturally to characterizations of new families of demand

[4] See Schmalensee (1981) and Aguirre, Cowan, and Vickers (2010).

functions that provide a parsimonious way of nesting existing ones, including most of those used in applied economics.[5]

A "firm's-eye view" is partial equilibrium by construction, of course. Nevertheless, it can provide the basis for understanding general equilibrium behavior. To demonstrate this, we show how our approach allows us to characterize the responses of outputs, prices, and product variety in the canonical model of international trade under monopolistic competition due to Krugman (1979). We show how the quantitative magnitude of the model's properties can be related to the assumed demand function through the lens of the implied demand manifold. Furthermore, we use our approach to derive new results for the case of heterogeneous firms, as in Melitz (2003), extended to general demands, as in Zhelobodko et al. (2012), Bertoletti and Epifani (2014), and Dhingra and Morrow (forthcoming). Following Dixit and Stiglitz (1977), we concentrate on the case of additively separable preferences, but our "firms'-eye perspective" can also be applied to other specifications of preferences, as in Dixit and Stiglitz (1993), Feenstra (2014), Bertoletti and Etro (2016), Bertoletti and Etro (2017), and Parenti, Ushchev, and Thisse (2017).

While the demand manifold is a theoretical construct, it also has potential empirical uses. In particular, it allows us to infer the parameters needed for comparative statics and counterfactual exercises, without estimating a demand function. We show that, given estimates of pass-through and markups, it is possible to back out the implied form of the demand manifold. With additional assumptions we can go further. Assuming that preferences are additively separable makes it possible to infer the implied income elasticities, while assuming parametric forms of demand opens the door toward quantifying the gains from trade.

The plan of the paper follows this route map. Section I introduces our new perspective on demand, and shows how the elasticity and convexity of demand condition comparative statics results. Section II shows how the demand manifold can be located in the space of elasticity and convexity, and explores how a wide range of demand functions, both old and new, can be represented by their manifold in a parsimonious way. Section III illustrates the usefulness of our approach by applying it to a canonical general-equilibrium model of international trade under monopolistic competition, and characterizing the implications of assumptions about functional form for the quantitative effects of exogenous shocks. Section IV turns to show how the demand manifold can be empirically estimated, and how it can be used for counterfactual analysis. Section V concludes, Appendix A gives some technical background and discusses some extensions, while online Appendix B gives proofs of all propositions, discusses some further extensions, and provides a glossary of terms used.

[5] Demand functions used in recent work that fit into our framework include the linear (Melitz and Ottaviano 2008), LES (Simonovska 2015), CARA (Behrens and Murata 2007), translog (Feenstra 2003), QMOR (Feenstra 2014), and Bulow-Pfleiderer (Atkin and Donaldson 2012). See Section IID and online Appendices B8 and B9.

## I. Demand Functions and Comparative Statics

### A. *A Firm's-Eye View of Demand*

A perfectly competitive firm takes the price it faces as given. Our starting point is the fact that a monopolistic or monopolistically competitive firm takes the demand function it faces as given. Observing economists will often wish to solve for the full general equilibrium of the economy, or to consider the implications of alternative assumptions about the structure of preferences (such as discrete choice, representative agent, homotheticity, separability, etc.); we will consider many such examples in later sections. By contrast, the firm takes all these as given and is concerned only with maximizing profits subject to the partial-equilibrium demand function it perceives. In this section, we consider the implications of this "firm's-eye view" of demand. For the most part we write the demand function in inverse form, $p = p(x)$, with the only restrictions that consumers' willingness to pay is continuous, three-times differentiable, and strictly decreasing in sales: $p'(x) < 0$.[6] It is sometimes convenient to switch to the corresponding direct demand function, the inverse of $p(x)$: $x = x(p)$, with $x'(p) < 0$.

As explained in the introduction, we express all our results in terms of the slope and curvature of demand, measured by two unit-free parameters, the elasticity $\varepsilon$ and convexity $\rho$ of the demand function:

$$\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0 \quad \text{and} \quad \rho(x) \equiv -\frac{xp''(x)}{p'(x)}. \tag{1}$$

These are not unique measures of slope and curvature, and our results could alternatively be presented in terms of other parameters, such as the convexity of the direct demand function, or the Kimball (1995) superelasticity of demand. Appendix A1 gives more details of these alternatives, and explains our preference for focusing on $\varepsilon$ and $\rho$.

Because we want to highlight the implications of alternative assumptions about demand, we assume throughout that marginal cost is constant.[7] Maximizing profits therefore requires that marginal revenue should equal marginal cost and should be decreasing with output. This imposes restrictions on the values of $\varepsilon$ and $\rho$ that must hold at a profit-maximizing equilibrium. From the first-order condition, a non-negative price-cost margin implies that the elasticity must be greater than one:

$$p + xp' = c \geq 0 \Rightarrow \varepsilon \geq 1. \tag{2}$$

From the second-order condition, marginal revenue $p + xp'$ decreasing with output implies that our measure of convexity must be strictly less then two:

$$2p' + xp'' < 0 \Rightarrow \rho < 2. \tag{3}$$

[6] We use "sales" throughout to denote consumption $x$, which in equilibrium equals the firm's output.

[7] Zhelobodko et al. (2012) show that variable marginal costs make little difference to the properties of models with homogeneous firms. In models of heterogeneous firms it is standard to assume that marginal costs are constant.

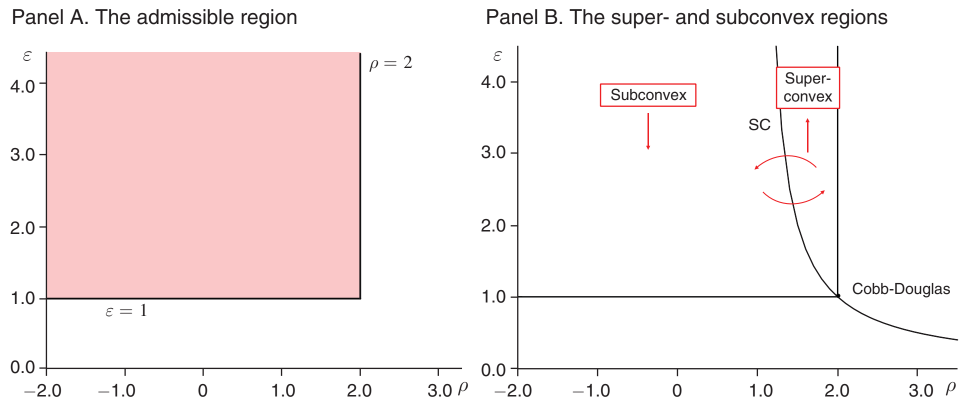


Figure 1. The Space of Elasticity and Convexity

These restrictions imply an admissible region in $\{\varepsilon, \rho\}$ space, as shown by the shaded region in Figure 1, panel A.[8] Consumers may be willing to consume outside the admissible region, but such points cannot represent the profit-maximizing equilibrium of a monopoly or monopolistically competitive firm.

## B. *The CES Benchmark*

In general, both $\varepsilon$ and $\rho$ vary with sales. The only exception is the case of CES preferences or iso-elastic demands:[9]

$$p(x) = \beta x^{-1/\sigma} \Rightarrow \varepsilon = \sigma, \; \rho = \rho^{CES} \equiv \frac{\sigma + 1}{\sigma} > 1. \tag{4}$$

Clearly this case is very special: both elasticity and convexity are determined by a single parameter, $\sigma$. Eliminating this parameter gives a relationship between $\varepsilon$ and $\rho$ that must hold for all members of the CES family: $\varepsilon = 1/(\rho - 1)$, or $\rho = (\varepsilon + 1)/\varepsilon$. This is illustrated by the curve labeled "SC" in Figure 1, panel B. Every point on this curve corresponds to a different CES demand function: firms always operate at that point irrespective of the values of exogenous variables. In this respect too the CES is very special, as we will see. The Cobb-Douglas special case corresponds to the point $\{\varepsilon, \rho\} = \{1, 2\}$, and so has the dubious distinction of being just on the boundary of both the first- and second-order conditions.

[8] The admissible region is $\{(\varepsilon, \rho) : 1 \leq \varepsilon < \infty \;\text{ and }\; -\infty < \rho < 2\}$. In the figures that follow, we illustrate the subset of the admissible region where $\varepsilon \leq 4.5$ and $\rho \geq -2.0$, since this is where most interesting issues arise and it is also consistent with the available empirical evidence. (Broda and Weinstein 2006, Soderbery 2015, and Benkovskis and Wörz 2014 estimate median elasticities of demand for imports of 3.7 or lower.) Note that the admissible region is larger in oligopolistic markets, since both boundary conditions are less stringent than (2) and (3). See Appendix A2 for details.

[9] It is convenient to follow the widespread practice of applying the "CES" label to the demand function in (4), though this only follows from CES preferences in the case of monopolistic competition, when firms assume they cannot affect the aggregate price index. The fact that CES demands are sufficient for constant elasticity is obvious. The fact that they are necessary follows from setting $-p(x)/xp'(x)$ equal to a constant $\sigma$ and integrating.

The CES case is important in itself but also because it is an important boundary for comparative statics results. Following Mrázová and Neary (forthcoming), we say that a demand function is "superconvex" at an arbitrary point if it is more convex at that point than a CES demand function with the same elasticity. Hence the eponymous SC curve in Figure 1, panel B, divides the admissible region in two: points to the right are strictly superconvex, points to the left are strictly subconvex, while all CES demand functions are both weakly superconvex and weakly subconvex. As we show in online Appendix B1, superconvexity also determines the relationship between demand elasticity and sales: the elasticity of demand increases in sales (or, equivalently, decreases in price), $\varepsilon_x \geq 0$, if and only if the demand function $p(x)$ is superconvex. So, $\varepsilon$ is independent of sales only along the SC locus, it increases with sales in the superconvex region to the right, and decreases with sales in the subconvex region to the left.[10] These properties imply something like the comparative-statics analogue of a phase diagram: the arrows in Figure 1, panel B, indicate the direction of movement as sales rise.[11]

## C. *Illustrating Comparative Statics Results*

We can use our diagram to illustrate some of the comparative statics results discussed in the introduction. The results themselves are not new, but illustrating them in a common framework provides new insights and sets the scene for our discussion of the implications of particular demand functions in Section II.

*Competition Effects and Relative Pass-Through: Superconvexity.*— Superconvexity itself determines both competition effects and relative pass-through: the effects of globalization and of cost changes, respectively, on firms' proportional profit margins. From the first-order condition, the relative markup or proportional profit margin $m \equiv (p - c)/c$ equals $-xp'/(p + xp')$, which is inversely related to the elasticity of demand: $m = 1/(\varepsilon - 1)$. Hence, if globalization reduces incumbent firms' sales in their home markets, it is associated with a higher elasticity and so a lower markup if and only if demand is subconvex. Similarly, an increase in marginal cost $c$, which other things equal must lower sales, is associated with a higher elasticity and so a lower markup, implying less than 100 percent pass-through, if and only if demands are subconvex:

$$\frac{d \log p}{d \log c} = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{2 - \rho} > 0 \quad \Rightarrow \tag{5}$$
$$\frac{d \log p}{d \log c} - 1 = -\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(2 - \rho)} \gtreqless 0.$$

[10] Many authors, including Marshall (1920), Dixit and Stiglitz (1977), and Krugman (1979), have argued that subconvexity is intuitively more plausible. (It is sometimes called "Marshall's Second Law of Demand." See online Appendix B19 for further discussion.) Moreover, subconvexity is consistent with much of the available empirical evidence on proportional pass-through, which suggests that it is less than 100 percent. See for example Gopinath and Itskhoki (2010), De Loecker et al. (2016), and our discussion in Section IVA. However, superconvexity cannot be ruled out either theoretically or empirically: as Zhelobodko et al. (2012) point out, some empirical studies find that entry or economic integration leads to higher markups. See, for example, Ward et al. (2002) and Badinger (2007).

[11] For most widely-used demand functions, the implied points in this space are always on one or other side of the SC curve. See Section II for further discussion and online Appendix B10 for a counterexample.

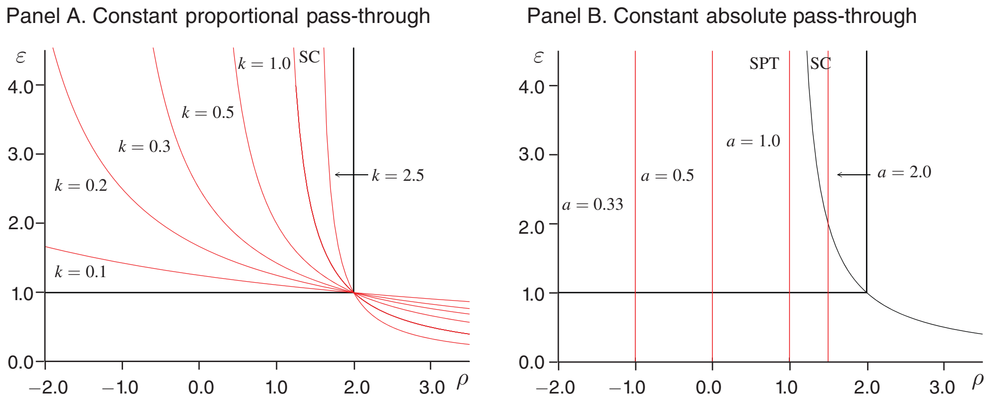


FIGURE 2. LOCI OF CONSTANT PASS-THROUGH

More generally, loci corresponding to $100k$ percent pass-through, i.e., $d\log p/d\log c = k$, are defined by[12]

$$\rho = 2 - \frac{1}{k}\frac{\varepsilon - 1}{\varepsilon}. \tag{6}$$

Figure 2, panel A, illustrates some of these loci for different values of $k$.

*Absolute Pass-Through: log-Convexity*.—The criterion for absolute or dollar-for-dollar pass-through from cost to price has been known since Bulow and Pfleiderer (1983). Differentiating the first-order condition $p + xp' = c$, we see that an increase in cost must raise price provided only that the second-order condition holds, which implies an expression for the effect of an increase in marginal cost on the absolute profit margin that is different from the proportional pass-through expression in (5):

$$\frac{dp}{dc} = \frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{dp}{dc} - 1 = \frac{\rho - 1}{2 - \rho} \gtreqless 0. \tag{7}$$

Hence we have what we call "super-pass-through," whereby the equilibrium price rises by more than the increase in marginal cost, if and only if $\rho$ is greater than one. More generally, loci corresponding to a pass-through coefficient of $a$ are defined by convexity values of $\rho = 2 - 1/a$. Figure 2, panel B, illustrates some of these loci for different values of $a$. The one corresponding to $a = 1$, labeled "SPT," divides the admissible region into subregions of sub- and super-pass-through. It corresponds to a log-linear direct demand function, which is less convex than the CES.[13] Hence superconvexity implies super-pass-through, but not the converse: in the region between the SPT and SC loci, pass-through is more than dollar-for-dollar but less than 100 percent. More generally, comparing panels A and B of Figure 2 shows

[12] This is a family of rectangular hyperbolas, all asymptotic to $\{\varepsilon, \rho\} = \{\infty, (2k-1)/k\}$ and $\{0, \infty\}$, and all passing through the Cobb-Douglas point $\{\varepsilon, \rho\} = \{1, 2\}$. We discuss this family further in Section IIE.

[13] Setting $\rho = 1$ implies a second-order ordinary differential equation $xp''(x) + p'(x) = 0$. Integrating this yields $p(x) = c_1 + c_2 \log x$, where $c_1$ and $c_2$ are constants of integration, which is equivalent to a log-linear direct demand function, $\log x(p) = \gamma + \delta p$.

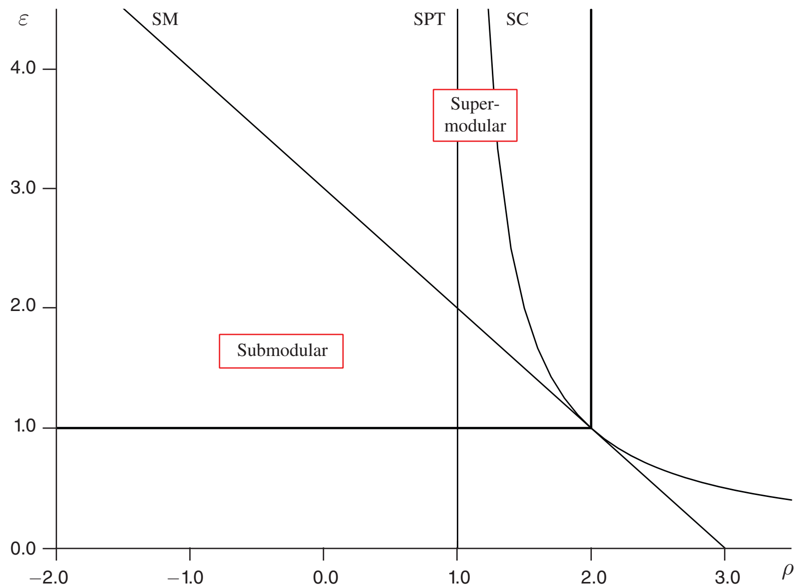


Figure 3. The Super- and Submodular Regions

that at any point the degree of absolute pass-through is greater than that of relative pass-through, and by more so the lower the elasticity; the implied relationship is: $a/k = \varepsilon/(\varepsilon - 1)$.

*Selection Effects: Supermodularity.*—A third criterion for comparative statics responses that we can locate in our diagram arises in models with heterogeneous firms, where firms choose between two alternative ways of serving a market, such as the choice between exports and foreign direct investment (FDI) as in Helpman, Melitz, and Yeaple (2004).[14] Mrázová and Neary (forthcoming) show that more efficient firms are sure to select into FDI only if their ex post profit function is *supermodular* in their own marginal cost $c$ and the iceberg transport cost they face $t$. Supermodularity holds if and only if the elasticity of marginal revenue with respect to sales is less than one, which in turn implies that the elasticity and convexity of demand sum to more than three.[15] When this condition holds, a 10 percent reduction in the marginal cost of serving a market raises sales by more than 10 percent, so more productive firms have a greater incentive to engage in FDI than in exports. This criterion defines a third locus in $\{\varepsilon, \rho\}$ space, as shown by the straight line labeled "SM" in Figure 3. Once again it divides the admissible region into two subregions,

[14] Mrázová and Neary (forthcoming) show that the same criterion determines selection effects in a number of other cases, including the choice between producing in the high-wage "North" or the low-wage "South" as in Antràs and Helpman (2004), and the choice of technique as in Bustos (2011). Related applications can be found in Spearot (2012, 2013).

[15] Let $\pi(c,t) \equiv \max_x[p(x) - tc]x$ denote the maximum operating profits which a firm with marginal production cost $c$ can earn facing an iceberg transport cost of accessing the market equal to $t$. When $\pi$ is twice differentiable, supermodularity implies that $\pi_{ct}$ is positive. By the envelope theorem, $\pi_c = -tx$. Hence, $\pi_{ct} = -x - t(dx/dt) = -x - tc/(2p' + xp'') = -x + x(\varepsilon - 1)/(2 - \rho)$. Writing revenue as $R(x) = xp(x)$, so marginal revenue is $R' = p + xp'$, the elasticity of marginal revenue (in absolute value) is seen to be: $-xR''/R' = (2 - \rho)/(\varepsilon - 1)$. The results in the text follow by inspection.

| Region | $\varepsilon_x > 0$ | $\frac{dp}{dc} > 0$ | $\varepsilon + \rho > 3$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | ✓ |
| 3 | | ✓ | |
| 4 | | ✓ | ✓ |
| 5 | ✓ | ✓ | ✓ |

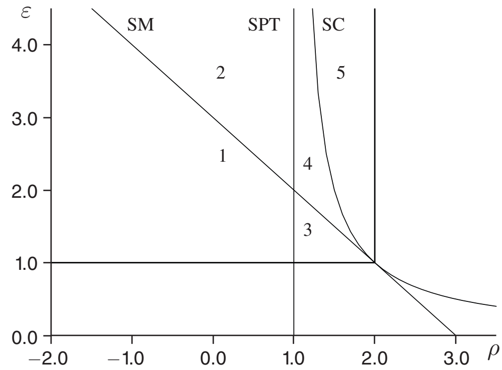


Figure 4. Regions of Comparative Statics

one where either the elasticity or convexity or both are high, so supermodularity prevails, and the other where the profit function is submodular. The locus lies everywhere below the superconvex locus, and is tangential to it at the Cobb-Douglas point. Hence, supermodularity always holds with CES demands. However, when demands are subconvex and firms are large (operating at a point on their demand curve with relatively low elasticity), submodularity prevails, and so the standard selection effects may be reversed.

### D. *Summary*

Figure 4 summarizes the results illustrated in this section. The three loci, corresponding to constant elasticity (SC), unit convexity (SPT), and unit elasticity of marginal revenue (SM), place bounds on the combinations of elasticity and convexity consistent with particular comparative statics outcomes. Of eight logically possible subregions within the admissible region, three can be ruled out because superconvexity implies both super-pass-through and supermodularity. From the figure it is clear that knowing the values of the elasticity and convexity of demand that a firm faces is sufficient to predict its responses to a wide range of exogenous shocks, including some of the classic questions posed in the introduction.

## II. The Demand Manifold

So far, we have shown how a wide range of comparative statics responses can be signed just by knowing the values of $\varepsilon$ and $\rho$ that a firm faces. Next we want to see how different assumptions about the form of demand determine these responses. To do this, Section IIA introduces our key innovation, the "demand manifold" corresponding to a particular demand function. We show that, in all cases other than the CES, the manifold is represented by a smooth curve in $(\varepsilon, \rho)$ space. Section IIB derives the conditions which guarantee that the manifold is invariant with respect to shifts in the demand function. Sections IIC and IID show how many widely-used demand functions can be parsimoniously represented by their demand manifolds, which provides a simple unifying principle for a very wide range of applications. Section IIE then shows how the demand manifold can be used to infer the comparative statics implications of a particular demand function, while Section IIF notes some demand functions whose manifolds are not invariant with respect to any of their parameters.

## A. *Demand Functions and Demand Manifolds*

Formally, we seek to characterize the set of values of the elasticity $\varepsilon$ and convexity $\rho$ that are consistent with a particular demand function $p_0 : x \mapsto p_0(x)$.

DEFINITION 1 (Definition of the Demand Manifold):

$$\Omega_{p_0} \equiv \left\{ (\varepsilon, \rho) : \varepsilon = -\frac{p_0(x)}{x p_0'(x)}, \quad \rho = -\frac{x p_0''(x)}{p_0'(x)}, \quad \forall x \in X_{p_0} \right\}, \tag{8}$$

*where the domain of* $p_0$ *is such that both output* $x$ *and price* $p$ *are non-negative*: $X_{p_0} \equiv \{x : x \geq 0 \text{ and } p_0(x) \geq 0\} \subset \mathbb{R}_{\geq 0}$.

We have already seen that the set $\Omega_{p_0}$, and hence the comparative statics responses implied by particular demand functions, are pinned down in one special case: facing a particular CES demand function, the firm is always at a single point in $(\varepsilon, \rho)$ space. Can anything be said more generally? The answer is "yes," as the following result shows.

PROPOSITION 1 (Existence of the Demand Manifold): *For every continuous, three-times differentiable, strictly-decreasing demand function,* $p_0(x)$, *other than the CES, the set* $\Omega_{p_0}$ *corresponds to a smooth curve in* $(\varepsilon, \rho)$ *space.*

The proof is in online Appendix B2. It proceeds by showing that, at any point on every demand function other than the CES, at least one of the functions $\varepsilon = \varepsilon(x)$ and $\rho = \rho(x)$ can be inverted to solve for $x$, and the resulting expression, denoted $x^{\varepsilon}(\varepsilon)$ and $x^{\rho}(\rho)$, respectively, substituted into the other function to give a relationship between $\varepsilon$ and $\rho$:

$$\varepsilon = \bar{\varepsilon}(\rho) \equiv \varepsilon[x^{\rho}(\rho)] \quad \text{or} \quad \rho = \bar{\rho}(\varepsilon) \equiv \rho[x^{\varepsilon}(\varepsilon)]. \tag{9}$$

We write this in two alternative ways, since at any given point only one may be well-defined, and, even when both are well-defined, one or the other may be more convenient depending on the context. The relationship between $\varepsilon$ and $\rho$ defined implicitly by (8) is not in general a function, since it need not be globally single-valued; but neither is it a correspondence, since it is locally single-valued. This is why we call it the "demand manifold" corresponding to the demand function $p_0(x)$. In the CES case, not covered by Proposition 1, we follow the convention that, corresponding to each value of the elasticity of substitution $\sigma$, the set $\Omega_{p_0}$ is represented by a point-manifold lying on the SC locus.

The first advantage of working with the demand manifold rather than the demand function itself is that it is located in $(\varepsilon, \rho)$ space, and so it immediately reveals the implications of assumptions made about demand for comparative statics. A second advantage, departing from the "firm's-eye view" that we have adopted so far, is that the manifold is often independent of exogenous parameters even though the demand function itself is not. Expressing this in the language of Chamberlin (1933), exogenous shocks typically shift the perceived demand curve, but they need not shift the corresponding demand manifold. We call this property "manifold invariance." When it holds, exogenous shocks lead only to movements along the manifold, not to shifts in it. As a result, it is particularly easy to make comparative statics predictions.

### B. *Manifold Invariance*

We wish to characterize the conditions under which manifold invariance holds. Clearly, the manifold cannot in most cases be invariant to changes in all parameters: even in the CES case, the point-manifold is not independent of the value of $\sigma$.[16] However, the CES point-manifold is invariant to changes in any parameter $\phi$ that affects the level term only; for ease of comparison with later functions, we write this in terms of both the inverse and direct CES demand functions:[17]

$$(10) \qquad \text{(a) } p(x,\phi) = \beta(\phi)x^{-1/\sigma} \quad \Leftrightarrow \quad \text{(b) } x(p,\phi) = \delta(\phi)p^{-\sigma}.$$

It is particularly convenient that the CES point-manifold is invariant with respect to variables (such as income or the prices of other goods) that are endogenous in general equilibrium and affect only the level term, whereas the parameter $\sigma$ with respect to which it is not invariant is a structural preference parameter. In the same way, as we show formally in Corollary 2, the manifold corresponding to any demand function turns out to be invariant with respect to its level parameter.

It is very desirable to have both necessary and sufficient conditions for a demand manifold to be invariant with respect to a particular parameter, and these are given by Proposition 2. Note that the proposition distinguishes between restrictions derived from inverse and direct demand functions (denoted (a) and (b), respectively). This was not necessary in the definition of the manifold in (8) and the proof of its existence in Proposition 1. However, it is needed here, because in general the responses of the elasticity and convexity of demand to a parameter change depend on whether price or quantity is assumed fixed.

PROPOSITION 2 (Manifold Invariance): *Assume that $\rho_x$ is nonzero. Then, the demand manifold is invariant with respect to a vector parameter $\phi$ if and only if both $\varepsilon$ and $\rho$ depend on $x$ and $\phi$ or on $p$ and $\phi$ through a common sub-function of either (a) $x$ and $\phi$; or (b) $p$ and $\phi$; i.e.:*

$$(11a) \qquad \varepsilon(x,\phi) = \tilde{\varepsilon}[F(x,\phi)] \quad \textit{and} \quad \rho(x,\phi) = \tilde{\rho}[F(x,\phi)];$$

*or*

$$(11b) \qquad \varepsilon(p,\phi) = \tilde{\varepsilon}[G(p,\phi)] \quad \textit{and} \quad \rho(p,\phi) = \tilde{\rho}[G(p,\phi)].$$

[16] Demand functions whose manifolds are invariant with respect to all demand parameters are relatively rare, though they include some well-known cases, including linear, Stone-Geary, CARA, and translog demands. See Section IID.

[17] These are equivalent, with $\beta(\phi) = \delta(\phi)^{1/\sigma}$.

The proof is in online Appendix B3.

To understand this result, note first that, just as Proposition 1 excluded the CES case where demand elasticity $\varepsilon$ is independent of $x$, so Proposition 2 excludes the case where demand convexity $\rho$ is independent of $x$. The class of demand functions that exhibits this property (which includes CES as a special case) is known as Bulow-Pfleiderer demands, and is considered separately in Section IID. Excluding this class, the proposition states that the only other case consistent with manifold invariance is where both $\varepsilon$ and $\rho$ satisfy a separability restriction, such that they depend on $\phi$ and on $x$ or $p$ via a common sub-function, $F(x, \phi)$ or $G(p, \phi)$. A useful corollary is where either $F$ or $G$ themselves is independent of $\phi$, which we can restate as follows.

COROLLARY 1: *The demand manifold is invariant with respect to a vector parameter $\phi$ if both elasticity $\varepsilon$ and convexity $\rho$ are independent of $\phi$.*

The next two subsections illustrate these results. Section IIC extends the CES demand functions from (10) in a nonparametric way and illustrates Corollary 1, while Section IID extends them parametrically by adding an additional power-law term and illustrates the general result in Proposition 2.

## C. *Multiplicatively Separable Demand Functions*

Our first result is that manifold invariance holds when the demand function is multiplicatively separable in $\phi$.

COROLLARY 2: *The demand manifold is invariant to shocks in a parameter $\phi$ if either (a) the inverse demand function or (b) the direct demand function is multiplicatively separable in $\phi$:*

$$p(x, \phi) = \beta(\phi)\tilde{p}(x); \tag{12a}$$

*or*

$$x(p, \phi) = \delta(\phi)\,\tilde{x}(p). \tag{12b}$$

The proof is in online Appendix B4, and relies on the convenient property that, with separability of this kind, both the elasticity and convexity are themselves invariant with respect to $\phi$, so Corollary 1 immediately applies.

This result has some important implications. First, when utility is additively separable, the inverse demand function for any good equals the marginal utility of that good times the inverse of the marginal utility of income. The latter is a sufficient statistic for all economy-wide variables that affect the demand in an individual market, such as aggregate income or the price index. A similar property holds for the direct demand function if the indirect utility function is additively separable (as in Bertoletti and Etro 2017), with the qualification that the indirect sub-utility functions depend on prices relative to income. (See online Appendix B4 for details.) Summarizing, we have the following result.

COROLLARY 3: *If preferences are additively separable, whether directly or indirectly, the demand manifold for any good is invariant to changes in aggregate variables (except for income, in the case of indirect additivity).*

Given the pervasiveness of additive separability in theoretical models of monopolistic competition, this is an important result, which implies that in many models the manifold is invariant to economy-wide shocks. We will see a specific application in Section III, where we apply our approach to the Krugman (1979) model of international trade with monopolistic competition.

A second implication of Corollary 2 comes by noting that, setting $\delta(\phi)$ in (12b) equal to market size $s$, yields the following.

COROLLARY 4: *The demand manifold is invariant to neutral changes in market size:* $x(p,s) = s\tilde{x}(p)$.

This corollary is particularly useful since it does not depend on the functional form of the individual demand function $\tilde{x}(p)$. An example that illustrates this is the logistic direct demand function: see online Appendix B5 for details.

Finally, a third implication of Corollary 2 is the dual of Corollary 4, and comes from setting $\beta(\phi)$ in (12a) equal to quality $q$.

COROLLARY 5: *The demand manifold is invariant to neutral changes in quality:* $p(x,q) = q\tilde{p}(x)$.

This implies that quality affects demand $x$ only through the quality-adjusted price $p/q$. Baldwin and Harrigan (2011) call this assumption "box-size quality": the consumer's willingness to pay for a single box of a good with quality level $q$ is the same as their willingness to pay for $q$ boxes of the same good with unit quality. Though special, it is a very convenient assumption, widely used in international trade, so it is useful that the comparative statics predictions of any such demand function are independent of the level of quality.

## D. *Bipower Demand Functions*

A second class of demand functions that exhibit manifold invariance comes from adding a second power-law term to the CES case (10), giving a "Bipower" or "Double CES" form. The corresponding manifolds can be written in closed form, as Proposition 3 shows.

PROPOSITION 3 (Bipower Demands): *The demand manifold is invariant to shocks in a parameter $\phi$ if either (a) the inverse demand function or (b) the direct demand function takes a bipower form:*

$$\text{(13a)} \qquad p(x,\phi) = \alpha(\phi)x^{-\eta} + \beta(\phi)x^{-\theta} \Leftrightarrow \bar{\rho}(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon;$$

$$(13b) \qquad x(p,\phi) = \gamma(\phi)p^{-\nu} + \delta(\phi)p^{-\sigma} \Leftrightarrow \bar{\rho}(\varepsilon) = \frac{\nu+\sigma+1}{\varepsilon} - \frac{\nu\sigma}{\varepsilon^2}.$$

The proof is in online Appendix B6. Clearly, the manifolds in (13a) and (13b) are invariant with respect to the level parameters $\{\alpha,\beta\}$ and $\{\gamma,\delta\}$, so changes in exogenous variables such as income or market size that only affect these parameters do not shift the manifold. (Hence we can suppress $\phi$ from here on.) Putting this differently, we need four parameters to characterize each demand function, but only two to characterize the corresponding manifold, which allows us to place bounds on the comparative statics responses reviewed in Section I.

However, the level parameters in (13a) and (13b) are also qualitatively important, as the following proposition shows.

PROPOSITION 4 (Superconvexity of Bipower Demands): *The bipower inverse demand functions in* (13a) *are superconvex if and only if both $\alpha$ and $\beta$ are positive. Similarly*, *the bipower direct demand functions in* (13b) *are superconvex if and only if both $\gamma$ and $\delta$ are positive.*

The proof is in online Appendix B7.[18] The two sets of parameters thus play very different roles. The power-law exponents $\{\eta,\theta\}$ and $\{\nu,\sigma\}$ determine the location of the manifold, whereas the level parameters $\{\alpha,\beta\}$ and $\{\gamma,\delta\}$ determine which "branch" of a particular manifold is relevant: the superconvex branch if they are both positive, the subconvex one if either of them is negative. (They cannot both be negative since both price and output are non-negative.) How this works is best understood by considering some special cases, that, as we will see, include some of the most widely-used demand functions in applied economics.

Two special cases of the bipower direct class (13b) are of particular interest.[19] The first, where $\nu = 0$, is the family of demand functions due to Pollak (1971).[20] The direct demand function is now a "translated" CES function of price: $x(p) = \gamma + \delta p^{-\sigma}$; while the demand manifold is a rectangular hyperbola: $\bar{\rho}(\varepsilon) = (\sigma+1)/\varepsilon$. Figure 5, panel A, illustrates some members of the Pollak family. They include many widely-used demand functions, including the CES ($\gamma = 0$), linear ($\sigma = -1$), Stone-Geary (or linear expenditure system (LES): $\sigma = 1$), and CARA (constant absolute risk aversion: the limiting case as $\sigma$ approaches zero).[21] Manifolds with $\sigma$ greater than one have two branches, one each in the sub- and

[18] This result has implications for the case where the direct demand function arises from aggregating across two groups with different CES preferences, with elasticities of substitution equal to $\nu$ and $\sigma$, respectively. Now the parameters $\gamma$ and $\delta$ depend inter alia on the weights of the two groups in the population, so both are positive and the market demand function must be superconvex.

[19] A third special case is the family of demand functions implied by the quadratic mean of order $r$ (QMOR) expenditure function introduced by Diewert (1976) and extended to monopolistic competition by Feenstra (2014). See online Appendix B8 for details on the Pollak, PIGL, and QMOR demand functions.

[20] Because $\nu$ and $\sigma$ enter symmetrically into (13b), it is arbitrary which is set equal to zero. For concreteness and without loss of generality we assume $\delta \neq 0$ and $\sigma \neq 0$ throughout.

[21] To show that CARA demands are a special case, rewrite the constants as $\gamma = \gamma' + \delta'/\sigma$ and $\delta = -\delta'/\sigma$, and apply l'Hôpital's Rule, which yields the CARA demand function $x = \gamma' + \delta' \log p$, $\delta' < 0$.

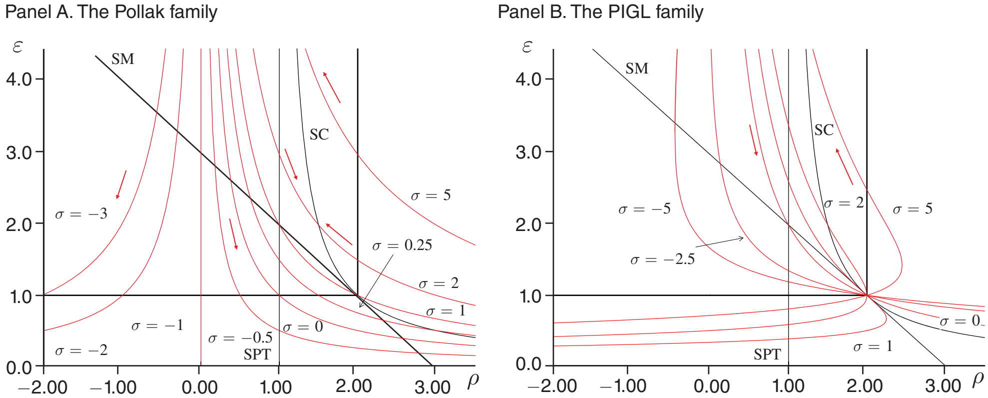


Figure 5. Demand Manifolds for Some Bipower Direct Demand Functions

superconvex regions, implying different directions of adjustment as sales increase, as indicated by the arrows.[22]

A second important special case of (13b) comes from setting $\nu = 1$. This gives the "PIGL" (price-independent generalized linear) system of Muellbauer (1975): $x(p) = \left(\gamma + \delta p^{1-\sigma}\right)/p$, which implies that expenditure $px(p)$ is a translated-CES function of price. From (13b), the manifold is given by: $\bar{\rho}(\varepsilon) = ((\sigma+2)\varepsilon - \sigma)/\varepsilon^2$. Figure 5, panel B, illustrates some PIGL demand manifolds. The best-known member of the PIGL family is the translog, $x(p) = (\gamma' + \delta' \log p)/p$, which is the limiting case as $\sigma$ approaches one so $\bar{\rho}(\varepsilon) = (3\varepsilon - 1)/\varepsilon^2$.[23] From the firm's perspective, this is consistent with both the Almost Ideal or "AIDS" model of Deaton and Muellbauer (1980), and the homothetic translog of Feenstra (2003). This class also includes Stone-Geary demands, the only case that is a member of both the Pollak and PIGL families (since $\nu = 0$ and $\sigma = 1$ are equivalent to $\nu = 1$ and $\sigma = 0$ in (13b)).

Just as the general bipower inverse and bipower direct demand functions in (13a) and (13b) are dual to each other, so also there are two important special cases of (13a) that are dual to the special cases of (13b) just considered. The first of these comes from setting $\eta$ in (13a) equal to zero, giving the inverse demand function $p(x) = \alpha + \beta x^{-\theta}$. This is the iso-convex or "constant pass-through" family of Bulow and Pfleiderer (1983), recently empirically implemented by Atkin and Donaldson (2012). The second important special case of (13a) comes from setting $\eta$ equal to one. This gives the "inverse PIGL" system, which is dual to the direct PIGL system considered earlier: expenditure $xp(x)$ is now a "translated-CES" function of sales: $p(x) = (\alpha + \beta x^{1-\theta})/x$. Further details about these demand functions and their manifolds are given in online Appendix B9.

[22] These branches correspond to the same value of $\sigma$ but to different values of $\gamma$ and/or $\delta$, and so to different demand functions. No bipower demand function as defined in Proposition 3 can be subconvex for some values of output and superconvex for others. Recalling Figure 1, panel B, this is only possible if the manifold is horizontal where it crosses the superconvexity locus. Online Appendix B10 gives an example of a demand function, the inverse exponential, that exhibits this property.

[23] To show this, rewrite the constants as $\gamma = \gamma' - \delta'/(1-\sigma)$ and $\delta = \delta'/(1-\sigma)$, and apply l'Hôpital's Rule, which yields the translog demand function.

### E. *Demand Manifolds and Comparative Statics*

It should be clear how the comparative statics implications of a given demand function can be illuminated by considering its demand manifold. To take a specific example, consider the Stone-Geary demand function (represented by the curves labeled $\sigma = 1$ in Figure 5, panel A, and $\sigma = 0$ in Figure 5, panel B). Referring back to Figures 2 and 4 in Section I, we can conclude without the need for any calculations that Stone-Geary demands are always subconvex, and that they imply less than absolute pass-through and supermodular profits for small firms but the opposite for large ones. Inspecting the figures shows that, qualitatively, these properties are similar to those of the CARA and translog demand functions (except for a qualification in the latter case discussed in the next paragraph). However, they are quite different from those of the CES on the one hand or the linear demand function on the other.

Comparing demand functions in terms of their manifolds can also draw attention to hitherto unsuspected results. An example, which is suggested by Figure 5, is that the translog is the only bipower demand function that is both subconvex and supermodular throughout the admissible region: see the curve labeled $\sigma = 1$ in Figure 5, panel B. We can go further and show that the translog is the only member of a broader class of demand functions, characterized in terms of their manifolds, that satisfies these conditions. We call the class in question a "contiguous bipower" manifold, since it expresses $\rho$ as a bipower function of $\varepsilon$, where the exponents are contiguous integers, $\kappa$ and $\kappa + 1$; this includes both bipower direct and bipower inverse demands, for which $\kappa$ equals $-2$ and zero, respectively.[24]

LEMMA 1: *The translog is the only demand function with a contiguous bipower manifold,* $\rho = a_1 \varepsilon^{\kappa} + a_2 \varepsilon^{\kappa+1}$, *where* $\kappa$ *is an integer, that is always both strictly subconvex and strictly supermodular in the interior of the admissible region.*

This is an attractive feature: the translog is the only demand function from a very broad family that allows for competition effects (so markups fall with globalization) but also implies that larger firms always serve foreign markets via FDI rather than exports.

Yet another use of the demand manifold is to back out demand functions with desirable properties. As an example, recall the discussion of proportional pass-through in Section IC. We saw there that the elasticity of pass-through, $d \log p / d \log c$, is constant and equal to $k$ $(k > 0)$ if and only if equation (6) holds. We can now see that this equation defines a family of demand manifolds for different values of $k$, as illustrated in Figure 2, panel A. Integrating it gives the implied demand function, which we call "CPPT" for "constant proportional of pass-through":[25]

$$p(x) = \frac{\beta}{x}\left(x^{\frac{k-1}{k}} + \gamma\right)^{\frac{k}{k-1}}. \tag{14}$$

[24] The proof is in online Appendix B11.

[25] Equation (14) is the solution to (6) when $k \neq 1$. When $k = 1$, (6) becomes $\rho = (\varepsilon + 1)/\varepsilon$, which, as we saw in Section IB, defines the family of point-manifolds for the CES case. Note that the CPPT manifold is a member of the contiguous bipower class, with $\kappa$ equal to $-1$, so Lemma 1 applies. See Appendix A3 for further details on the CPPT demand function.

This demand function appears to be new and is likely to have many uses in applied work. We will give an illustration in Section IVA. In the special case where $k = 1/2$, the CPPT demand function is identical to the Stone-Geary, both with manifolds given by $\rho = 2/\varepsilon$. This yields the result that, with Stone-Geary demands, all firms pass through exactly 50 percent of cost increases to consumers.

### F. *Demand Functions that Are Not Manifold-Invariant*

In the rest of the paper we concentrate on the demand functions introduced here which have manifolds that are invariant with respect to at least some parameters. However, even in more complex cases when the demand manifold has the same number of parameters as the demand function, it typically provides an economy of information by highlighting which parameters matter for comparative statics. Online Appendix B12 presents two examples of this kind that nest some important cases, such as the "Adjustable Pass-Through" (APT) demand function of Fabinger and Weyl (2012).

## III. Monopolistic Competition in General Equilibrium

To illustrate the power of the approach we have developed in previous sections, we turn next to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general-equilibrium model of monopolistic competition, where countries are symmetric and trade is unrestricted.[26] Following Krugman (1979) and a large subsequent literature, we model globalization as an increase in the number of countries in the world economy. On the consumer side, we assume that preferences are symmetric, and that the elasticity of demand for a good depends only on the level of consumption of that good. From Goldman and Uzawa (1964), this is equivalent to assuming additively separable preferences as in Dixit and Stiglitz (1977) and Krugman (1979):

$$U = F\left[\int_0^N u\{x(\omega)\}\, d\omega\right], \; F' > 0, \; u'(x) > 0, \; u''(x) < 0. \tag{15}$$

We begin with the effects of globalization on industry equilibrium, first in Section IIIA with homogeneous firms as in Krugman (1979), and then in Section IIIB with heterogeneous firms as in Melitz (2003). Finally in Section IIIC we look at the effects of globalization on welfare.

### A. *Globalization with Homogeneous Firms*

Symmetric demands and homogeneous firms imply that we can dispense with firm subscripts from the outset. Industry equilibrium requires that firms maximize profits by choosing output $y$ to set marginal revenue MR equal to marginal cost

[26] The effects of changes in trade costs are considered in Mrázová and Neary (2014).

MC, and that profits are driven to zero by free entry (so average revenue AR equals average cost AC):

$$\text{(16)} \qquad \text{Profit Maximization (MR = MC):} \quad p = \frac{\varepsilon(x)}{\varepsilon(x) - 1} c,$$

$$\text{(17)} \qquad \text{Free Entry (AR = AC):} \quad p = \frac{f}{y} + c.$$

The model is completed by market-clearing conditions for the goods and labor markets:

$$\text{(18)} \qquad \text{Goods-Market Equilibrium (GME):} \quad y = kLx,$$

$$\text{(19)} \qquad \text{Labor-Market Equilibrium (LME):} \quad L = n(f + cy).$$

Here $L$ is the number of worker/consumers in each country, each of whom supplies one unit of labor and consumes an amount $x$ of every variety; $k$ is the number of identical countries; and $n$ is the number of identical firms in each, all with total output $y$, so $N = kn$ is the total number of firms in the world. Since all firms are single-product by assumption, $N$ is also the total number of varieties available to all consumers.

Equations (16) to (19) comprise a system of four equations in four endogenous variables, $p$, $x$, $y$, and $n$, with the wage rate set equal to one by choice of numéraire. To solve for the effects of globalization, an increase in the number of countries $k$, we totally differentiate the equations, using "hats" to denote logarithmic derivatives, so $\hat{x} \equiv d\log x, x \neq 0$:

$$\text{(20)} \qquad \text{MR = MC:} \quad \hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(\varepsilon - 1)} \hat{x},$$

$$\text{(21)} \qquad \text{AR = AC:} \quad \hat{p} = -(1 - \omega)\hat{y},$$

$$\text{(22)} \qquad \text{GME:} \quad \hat{y} = \hat{k} + \hat{x},$$

$$\text{(23)} \qquad \text{LME:} \quad 0 = \hat{n} + \omega\hat{y}.$$

Consider first the MR $=$ MC equilibrium condition, equation (20). Clearly $p$ and $x$ move together if and only if $\varepsilon + 1 - \varepsilon\rho > 0$, i.e., if and only if demand is subconvex. This reflects the property noted in Section IB: higher sales are associated with a higher proportional markup, $(p - c)/c$, if and only if they imply a lower elasticity of demand. As for the free-entry condition, equation (21), it shows that the fall in price required to maintain zero profits following an increase in firm output is greater the smaller is $\omega \equiv cy/(f + cy)$, the share of variable in total costs, which is an inverse measure of returns to scale. This looks like a new parameter but in equilibrium it is not. It equals the ratio of marginal cost to price, $c/p$, which because of profit maximization equals the ratio of marginal revenue to price $(p + xp')/p$, which in turn is a monotonically increasing transformation of the elasticity of demand $\varepsilon$: $\omega = c/p = (p + xp')/p = (\varepsilon - 1)/\varepsilon$. Similarly, equation (23) shows that the fall in the number of firms required to maintain full employment following an increase in firm output is greater the larger is $\omega$. It follows by inspection that all four equations depend only on two parameters, which implies the following.

LEMMA 2: *With additive separability, the local comparative statics responses of the symmetric monopolistic competition model to a globalization shock depend only on $\varepsilon$ and $\rho$.*

Lemma 2 implies that the comparative statics results can be directly related to the demand manifold. To see this in detail, solve for the effects of globalization on outputs, prices, and the number of firms in each country:

$$(24) \quad \hat{y} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(2 - \rho)}\hat{k}, \quad \hat{p} = -\frac{1}{\varepsilon}\hat{y} = -\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon^2(2 - \rho)}\hat{k}, \quad \hat{n} = -\frac{\varepsilon - 1}{\varepsilon}\hat{y}.$$

(Details of the solution are given in online Appendix B13.) The signs of these depend solely on whether demands are sub- or superconvex, i.e., whether $\varepsilon + 1 - \varepsilon\rho$ is positive or negative. With subconvexity we get what Krugman (1979) called "sensible" results: globalization prompts a shift from the extensive to the intensive margin, with fewer but larger firms in each country, as firms move down their average cost curves and prices of all varieties fall. With superconvexity, all these results are reversed.[27] The CES case, where $\varepsilon + 1 - \varepsilon\rho = 0$, is the boundary one, with firm outputs, prices, and the number of firms per country unchanged. The only effects that hold irrespective of the form of demand are that consumption per head of each variety falls and the total number of varieties produced in the world and consumed in each country rises:

$$(25) \quad \hat{x} = -\frac{1}{2 - \rho}\frac{\varepsilon - 1}{\varepsilon}\hat{k} < 0, \quad \hat{N} = \frac{(\varepsilon - 1)^2 + (2 - \rho)\varepsilon}{\varepsilon^2(2 - \rho)}\hat{k} > 0.$$

Qualitatively these results are not new. The new feature that our approach highlights is that their quantitative magnitudes depend only on two parameters, $\varepsilon$ and $\rho$, the same ones on which we have focused throughout. Hence the results in (24) and (25) can be directly related to the demand manifold from Section II.

To illustrate how this works, Figure 6 gives the quantitative magnitudes of changes in the two variables that matter most for welfare: prices and the number of varieties. In each panel, the vertical axis measures the proportional change in either $p$ or $N$ following a unit increase in $k$ as a function of the elasticity and convexity of demand. The three-dimensional surfaces shown are independent of the functional form of demand, so we can combine them with the results on demand manifolds from Section II to read off the quantitative effects of globalization implied by different assumptions about demand. We know already from equations (24) and (25) that prices fall if and only if demand is subconvex and that product variety always rises.

[27] See Neary (2009) and Zhelobodko et al. (2012).

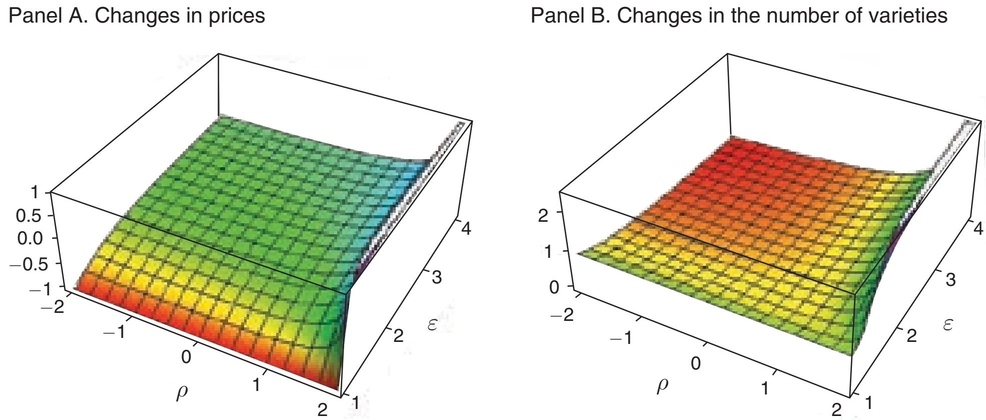


Figure 6. The Effects of Globalization

The figures show in addition that less elastic demand implies greater falls in prices and larger increases in variety, except when demand is highly convex;[28] while more convex demand always implies greater increases in both prices and variety.

To summarize this subsection, Lemma 2 implies that the demand manifold is a sufficient statistic for the effects of globalization on industry equilibrium in the Krugman (1979) model, just as it is for the comparative statics results discussed in Section I. Moreover, as in Section IIE, given a particular demand function, we can immediately infer its implications for the comparative statics of globalization by combining its demand manifold with Figure 6.

## B. *Heterogeneous Firms*

The case of homogeneous firms is of independent interest, and also provides a key reference point for understanding the comparative statics of a model with heterogeneous firms and general demands. Consider the same model as before, except that now firms differ in their marginal costs $c$, which, as in Melitz (2003), are drawn from an exogenous distribution $g(c)$, with support on $[\underline{c}, \bar{c}]$. The maximum operating profit that a firm can earn varies inversely with its own marginal cost $c$. Through the inverse demand function $p(y, \lambda, k)$, it also depends negatively on the marginal utility of income, $\lambda$, and positively on the size of the global economy $k$:

$$\pi(\underset{-}{c}, \underset{-}{\lambda}, \underset{+}{k}) \equiv \max_{y}\left[p(y, \lambda, k) - c\right] y. \tag{26}$$

A key implication of this specification is that, in monopolistic competition, where individual firms are infinitesimal relative to the industry, $\lambda$ is endogenous to the industry, but exogenous to firms, and so can be interpreted as a measure of the degree of competition each firm faces.[29]

[28] $\hat{p}/\hat{k}$ is increasing in $\varepsilon$ if and only if $\rho < 1 + 2/\varepsilon$, and $\hat{N}/\hat{k}$ is decreasing in $\varepsilon$ if and only if $\rho < 2/\varepsilon$.

[29] This specification is also consistent with a much broader class of preferences than additive separability, which Pollak (1972) calls "generalized additive separability." See Mrázová and Neary (forthcoming) for further discussion.

With homogeneous firms, equation (17) in Section IIIA gives a free-entry condition that is common to all firms. With heterogeneous firms, this must be replaced by two conditions. First is the zero-profit condition for cutoff firms, which requires that their operating profits equal the common fixed cost $f$:

$$\pi(c_0, \lambda, k) = f. \tag{27}$$

This determines the cutoff cost $c_0$ as a function of $\lambda$ and $k$. Second is the zero-expected-profit condition for all firms. A potential entrant bases its entry decision on the value $v(c, \lambda, k)$ that it expects to earn; firm value is zero for firms that get a high-cost draw and equals operating profits less fixed costs otherwise. Equilibrium requires that the expected value of a firm, $\bar{v}(\lambda, k)$, equal the sunk cost of entering the industry $f_e$:

$$\bar{v}(\lambda, k) \equiv \int_{\underline{c}}^{\bar{c}} v(c, \lambda, k) g(c)\, dc = f_e, \tag{28}$$

where

$$v(c, \lambda, k) \equiv \max[0, \pi(c, \lambda, k) - f].$$

Expected profits are conditional on incurring the sunk cost of entry, not conditional on actually entering, and so they do not depend directly on the cutoff $c_0$. Equation (28) thus determines the level of competition as a function of the size of the world economy $k$.

We can now derive the effects of globalization on the profile of profits across firms. Combining the profit function and equation (28) gives

$$\hat{\pi} = \frac{k\pi_k}{\pi}\hat{k} + \frac{\lambda\pi_\lambda}{\pi}\hat{\lambda} = \Bigg(\underbrace{\frac{k\pi_k}{\pi}}_{\text{(M)}} - \underbrace{\frac{\lambda\pi_\lambda}{\pi}\frac{\bar{v}}{\lambda\bar{v}_\lambda}\frac{k\bar{v}_k}{\bar{v}}}_{\text{(C)}}\Bigg)\hat{k}. \tag{29}$$

This shows that globalization has a market-size effect, given by (M), which tends to *raise* each firm's profits. In addition, it has a competition effect, given by (C): because *all* firms' profits rise at the initial level of competition, the latter must increase to ensure that expected profits remain equal to the fixed cost of entry; this in turn tends to *reduce* each firm's profits. The net outcome is indeterminate in general. However, with additive separability, equation (29) takes a particularly simple form (see Appendix A4 for details):

$$\hat{\pi} = \left(1 - \tfrac{\varepsilon}{\bar{\varepsilon}}\right)\hat{k} \quad \text{where} \quad \bar{\varepsilon} \equiv \int_{\underline{c}}^{\bar{c}} \frac{v(c, \lambda, k)}{\bar{v}(\lambda, k)} \varepsilon(c) g(c)\, dc. \tag{30}$$

Here $\bar{\varepsilon}$ is the profit-weighted average elasticity of demand across all firms, which we can interpret as the elasticity faced by the average firm. Thus the market-size effect is one-for-one (given $\lambda$, all firms' profits increase proportionally with $k$), while the competition effect is greater than one if and only if the elasticity a firm faces is

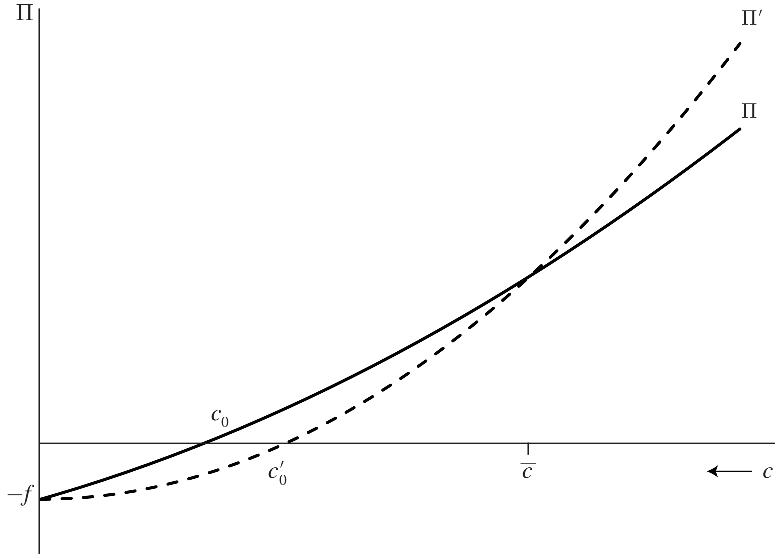


Figure 7. Effects of Globalization on Firm Profits and Selection with Subconvex Demand

greater than the average elasticity. The implications for the response of profits across firms are immediate, recalling that firms face an elasticity of demand that falls with their output if and only if demands are subconvex.

PROPOSITION 5: *With additive separability*, *globalization pivots the profile of profits across firms around the average firm; if and only if demands are subconvex*, *profits rise for firms above the average*, *and by more the larger a firm's initial sales.*

As in Section 4.1 of Melitz (2003), globalization leaves the profits of all firms unchanged in the CES case (where $\varepsilon = \bar{\varepsilon} = \sigma$ for all firms). By contrast, in the realistic case when demand is subconvex, the elasticity of demand is smaller for firms with above-average output, and so the outcome exhibits a strong "Matthew Effect" ("to those who have, more shall be given"). This is illustrated in Figure 7, where the solid locus $\Pi$ denotes the initial profile of profits across firms, while the dashed locus $\Pi'$ denotes the post-globalization profile when demand is subconvex.[30] The market-size effect dominates for larger firms, so they expand; the competition effect dominates for smaller firms, so they contract, and some (those at or just to the right of the initial cutoff cost level $c_0$) exit;[31] as a result, the average productivity of active exporters rises. All these results are reversed when demands are superconvex: now larger firms face higher elasticities of demand, so their profits fall, whereas those of smaller firms rise, and globalization encourages entry of less efficient firms.

[30] Marginal cost $c$ is increasing from right to left along the horizontal axis. It can be checked that profits are decreasing and convex in $c$.

[31] To solve for the effect of globalization on the extensive margin, we can use (30) to evaluate the change in the cutoff marginal cost defined by (27): $\hat{c}_0/\hat{k} = (1 - \varepsilon_0/\bar{\varepsilon})/(\varepsilon_0 - 1)$, where $\varepsilon_0 \equiv \varepsilon(c_0)$ is the elasticity faced by firms at the cutoff. Such firms have the lowest sales of all active firms and so, when demands are subconvex, they face the highest elasticity: $\varepsilon_0 > \bar{\varepsilon}$. Hence the competition effect dominates and the least efficient firms exit.

In the same way we can solve for the effects of globalization on the intensive margin. As shown in Appendix A4, the changes in the profiles of firm outputs and prices are given by

$$\hat{y} = \Bigg[\underbrace{\frac{\bar{\varepsilon} + 1 - \bar{\varepsilon}\bar{\rho}}{\bar{\varepsilon}(2 - \bar{\rho})}}_{*} + \tfrac{1}{\bar{\varepsilon}}\left(\frac{\bar{\varepsilon} - 1}{2 - \bar{\rho}} - \frac{\varepsilon - 1}{2 - \rho}\right)\Bigg]\hat{k}, \tag{31}$$

$$\hat{p} = \Bigg[\underbrace{-\frac{\bar{\varepsilon} + 1 - \bar{\varepsilon}\bar{\rho}}{\bar{\varepsilon}^2(2 - \bar{\rho})}}_{*} - \tfrac{1}{\bar{\varepsilon}}\left(\frac{\bar{\varepsilon} - 1}{\bar{\varepsilon}(2 - \bar{\rho})} - \frac{\varepsilon - 1}{\varepsilon(2 - \rho)}\right)\Bigg]\hat{k}. \tag{32}$$

These changes in output and price for each firm have two components. The first, denoted by $*$, equals the change for the average firm, which is the same as the change for all firms in the homogeneous-firms case (given by (24)). Hence, for the average firm, output rises and price falls if and only if the demand it faces is subconvex. Figure 6, panel A, therefore illustrates the change in the average firm's price, so, as in Section IIIA, we can evaluate this by combining the figure with the appropriate demand manifold. The second component is a correction factor that adjusts for the differences between the individual firm and the average firm. Its sign depends on the difference between both the elasticity and convexity of the individual firm and those of the average firm. For example, if demand is subconvex, then outputs of above-average firms tend to rise relative to the average firm, and to rise by more the larger the firm; while outputs of below-average firms tend to fall relative to the average firm, and to fall by more the smaller the firm.[32] Similar considerations apply to the change in prices.

### C. *Globalization and Welfare*

The final application of the manifold we consider is to the effects of globalization on welfare. It is clear that the demand parameters summarized by the manifold are an important component of calculating the gains from globalization, but it is also clear that they cannot be a sufficient statistic for welfare change in general. At the very least, if firms are heterogeneous, we also need to know one or more parameters of the productivity distribution. However, the manifold is a sufficient statistic for the gains from trade in some cases: specifically, when the distribution of firm productivities is degenerate, so firms are homogeneous, and when the functional form of the sub-utility function is restricted in ways to be explained below. So, to highlight the role of the manifold, we return in this subsection to the case of homogeneous firms as in Section IIIA. As a benchmark, this case is of great interest

[32] Recalling footnote 15, the correction factor for outputs depends on the difference between the inverse elasticity of marginal revenue of the individual firm and that of the average firm. The exact condition for the change in output with globalization to be increasing in firm size is $(2-\rho)\varepsilon_y + (\varepsilon - 1)\rho_y < 0$. When demand is subconvex (so $\varepsilon_y < 0$), this condition holds for almost all the demand functions discussed in Section IID, including all members of the PIGL and Bulow-Pfleiderer families (trivially for the latter since $\rho_y = 0$), and almost all members of the Pollak family. However, this tendency could be reversed if $\rho_y$ were positive and sufficiently large.

in itself. It also gives a lower bound to the gains from trade in an otherwise identical model with firm heterogeneity, at least with CES preferences, as shown empirically and theoretically by Balistreri, Hillberry, and Rutherford (2011) and Melitz and Redding (2015).[33]

To quantify the welfare effects of globalization, we assume as in previous sections that preferences are additively separable. With homogeneous firms, symmetric preferences, and no trade costs, the overall utility function (15) becomes: $U = F[Nu(x)]$. So, welfare depends on the extensive margin of consumption $N$ times the utility of the intensive margin $x$. Using the budget constraint to eliminate $x$, we can write the change in utility in terms of its income equivalent $\hat{Y}$ (see online Appendix B14 for details):

$$\hat{Y} = \frac{1 - \xi}{\xi}\hat{N} - \hat{p}. \tag{33}$$

Here $\xi(x) \equiv xu'(x)/u(x)$ is the elasticity of the sub-utility function $u(x)$ with respect to consumption. We thus have a clear division of roles between three preference parameters: on the one hand, as we saw in Section IIIA, $\varepsilon$ and $\rho$ determine the effects of globalization on the two variables, number of varieties, $N$, and prices, $p$, that affect consumers directly; on the other hand, $\xi$ determines the relative importance of $N$ and $p$ in affecting welfare. It is clear from (33) that $\xi$ must lie between zero and one if preferences exhibit a taste for variety. (See also Vives 1999.) Moreover, $\xi$ is an inverse measure of preference for variety, since welfare rises more slowly with $N$ the higher is $\xi$.

Next, we can substitute for the changes in prices and number of varieties from equations (24) and (25) in Section IIIA into (33) to obtain an explicit expression for the gain in welfare in terms of preference and demand parameters only:

$$\hat{Y} = \frac{1}{\xi\varepsilon}\left[1 - \left(\xi - \frac{\varepsilon - 1}{\varepsilon}\right)\frac{\varepsilon - 1}{2 - \rho}\right]\hat{k}. \tag{34}$$

Now there are three sufficient statistics for the change in welfare, only one of which has an unambiguous effect. The gains from globalization are always decreasing in $\xi$ : unsurprisingly, consumers gain more from a proliferation of countries, and hence of products, the greater their taste for variety. By contrast, the gains from globalization depend ambiguously on both $\varepsilon$ and $\rho$. Of course, the values of the three key parameters do not in general vary independently of each other, but without further assumptions we cannot say much about how they vary together.

One case where equation (34) simplifies dramatically is when preferences are CES, so $u(x) = (\sigma/(\sigma - 1))\beta x^{(\sigma-1)/\sigma}$. Now the elasticity of utility $\xi$ equals $(\sigma - 1)/\sigma$, while $\varepsilon$ and $\rho$ equal $\sigma$ and $(\sigma + 1)/\sigma$, respectively, as we have already seen in Section IB. Substituting these values into (34), the gains from

[33] By "otherwise identical" we mean with the same structural parameters except a nondegenerate distribution of firm productivities. If instead the comparison is carried out holding constant the elasticity of trade, then the gains from trade are the same in homogeneous and heterogeneous firms models as shown by Arkolakis, Costinot, and Rodríguez-Clare (2012).

globalization, $\hat{Y}/\hat{k}$, reduce to $1/(\sigma - 1)$, exactly the expression found for the gains from trade in a range of CES-based models by Arkolakis, Costinot, and Rodríguez-Clare (2012).

The key feature of the CES case is that the elasticities of utility and demand are directly related, without the need to specify any parameters. To move beyond the CES case, we would like to be able to express the elasticity of utility $\xi$ as a function of $\varepsilon$ and $\rho$ only. If this function is independent of parameters, then we can locate equation (34) in $(\varepsilon, \rho)$ space, and use the results of Section II to relate it to the underlying demand function. To see when this can be done, recall that the demand manifold relates $\varepsilon$, $\rho$, and the non-invariant parameter $\phi$, which in general is vector-valued. To this can be added a second equation, which we call the "utility manifold," that relates $\xi$, $\varepsilon$, and $\phi$.[34] We thus have two equations in $3 + m$ unknowns, where $m$ is the dimension of $\phi$. Clearly, the demand manifold, and the space of $\{\varepsilon, \rho\}$ that it highlights, is particularly useful when $m$ equals one, since then we can eliminate $\phi$. In that case we can express both the elasticity of utility, $\xi$, and hence, using (34), the gains from globalization, $\hat{Y}/\hat{k}$, as functions of $\varepsilon$ and $\rho$ only. We can summarize these results in a way that brings out the parallel with Lemma 2 in Section IIIA.

LEMMA 3: *With additive separability, the gains from globalization in the symmetric monopolistic competition model depend only on* $\varepsilon$ *and* $\rho$ *if and only if* $\phi$, *the vector of non-invariant parameters in the utility and demand manifolds, is of dimension less than or equal to one.*

To see the usefulness of this, we consider two of the families of demand functions discussed in Section II, whose manifolds depend on only a scalar non-invariant parameter.

*Globalization and Welfare with Bulow-Pfleiderer Preferences.*—The first example we consider is that of Bulow-Pfleiderer demands, given by the demand function (13a) in Section IID. Assuming that preferences are additively separable, we can integrate that function to obtain the corresponding sub-utility function, which also takes a bipower form:[35]

$$u(x) = \alpha x + \frac{1}{1-\theta}\beta x^{1-\theta}. \tag{35}$$

From this we can calculate the utility manifold, which gives the elasticity of utility $\xi$ as a function of $\varepsilon$ and the non-invariant parameter $\theta$, and then use the demand

[34] In an earlier version of this paper, Mrázová and Neary (2013), we gave more details of this equation and its geometric representation. Its derivation parallels that of the demand manifold. Recall that the demand manifold is derived by eliminating consumption $x$ from the expressions for the elasticity and curvature of the demand function, $\varepsilon(x, \phi)$ and $\rho(x, \phi)$, to obtain a smooth curve in $(\varepsilon, \rho)$ space. In the same way, eliminating $x$ from the expressions for the elasticity and curvature of the sub-utility function, $\xi(x, \phi)$ and $\varepsilon(x, \phi)$, yields a smooth curve in $\{\xi, \varepsilon\}$ space, whose properties are analogous to those of the demand manifold.

[35] It is natural to set the constant of integration to zero, which implies that $u(0) = 0$. We return to this issue in the next subsection.

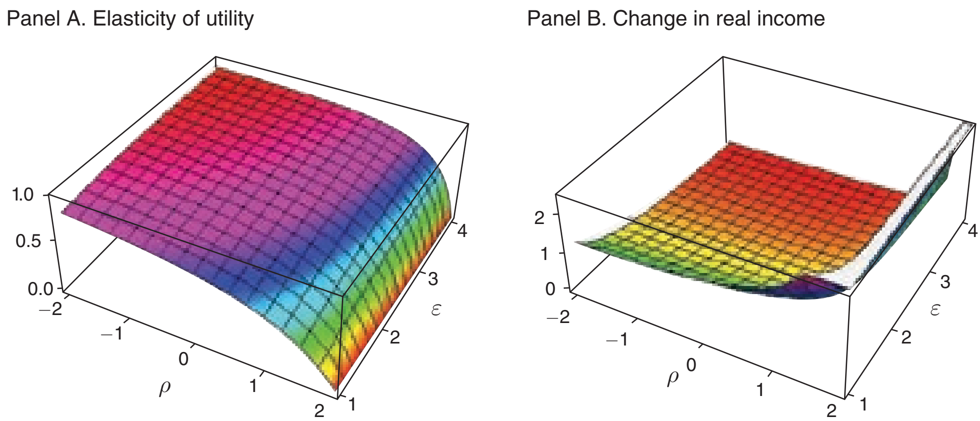


Figure 8. Globalization and Welfare: Bulow-Pfleiderer Preferences

manifold from Section IID, $\rho = \theta + 1$, to solve for $\xi$ as a function of $\varepsilon$ and $\rho$ (details are in online Appendix B16):

$$\xi(\varepsilon, \theta) = \frac{(1-\theta)\varepsilon}{(1-\theta)\varepsilon + 1} \Rightarrow \xi(\varepsilon, \rho) = \frac{(2-\rho)\varepsilon}{(2-\rho)\varepsilon + 1}. \tag{36}$$

Clearly $\xi$ always lies between zero and one when $\varepsilon$ and $\rho$ are in the admissible region. Figure 8, panel A, illustrates the second of these expressions, while substituting into equation (34) allows us to express the change in real income as a function of $\varepsilon$ and $\rho$ only, as illustrated in Figure 8, panel B. As panel A illustrates, the elasticity of utility is increasing in the elasticity of demand, and decreasing in convexity: there is a greater taste for variety at high $\rho$; while panel B shows that the gains from globalization are always positive, are decreasing in $\varepsilon$, and increasing in $\rho$.

*Globalization and Welfare with Pollak Preferences.*—The second example we consider is the Pollak demand function from Section IID. The welfare implications of this specification are sensitive to how we normalize the sub-utility function. To highlight the contrast with the Bulow-Pfleiderer case in the previous subsection, we focus in the text on the case considered by Pollak (1971) and Dixit and Stiglitz (1977). This derives the sub-utility function from the demand function without imposing a constant of integration, implying that $u(0)$ is nonzero: consumers gain from the presence of more varieties even if they do not consume them, an outcome whose plausibility was defended by Dixit and Stiglitz (1979).[36] This gives the following sub-utility function:

$$u(x) = \frac{\beta}{\sigma - 1}(\sigma x + \zeta)^{\frac{\sigma-1}{\sigma}}. \tag{37}$$

[36] In online Appendix B16 we consider an alternative specification, due to Pettengill (1979), which imposes the restriction that $u(0) = 0$, and yields different results.

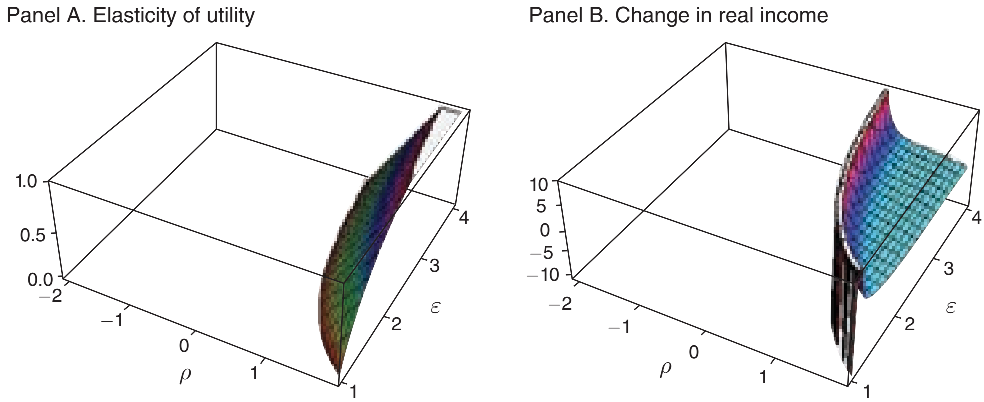


FIGURE 9. GLOBALIZATION AND WELFARE: POLLAK PREFERENCES

Relative to the Pollak demand function in Section IID, it is convenient to redefine the constants as $\zeta \equiv -\gamma\sigma$ and $\beta \equiv (\delta/\sigma)^{1/\sigma}$. (See online Appendix B16 for details.)

As in the Bulow-Pfleiderer case, we can calculate the utility manifold, giving $\xi$ as a function of $\varepsilon$ and the non-invariant parameter $\sigma$, and then use the demand manifold from Section IID, $\bar{\rho}(\varepsilon) = (\sigma + 1)/\varepsilon$, to solve for $\xi$ as a function of $\varepsilon$ and $\rho$:

$$(38) \qquad \xi(\varepsilon, \sigma) = \frac{\sigma - 1}{\varepsilon} \quad \Rightarrow \quad \xi(\varepsilon, \rho) = \frac{\varepsilon\rho - 2}{\varepsilon}.$$

Since only values of $\xi$ between zero and one are consistent with a preference for variety, we restrict attention to the range $2/\varepsilon < \rho < 1 + 2/\varepsilon$.[37] Within this range, the behavior of the elasticity of utility is the opposite to that in the Bulow-Pfleiderer case.[38] Figure 9 illustrates how the elasticity of utility and the gains from globalization vary with $\varepsilon$ and $\rho$ in this case. The contrast with the Bulow-Pfleiderer case in Figure 8 could hardly be more striking. Panel A shows that the elasticity of utility is now increasing in *both* $\varepsilon$ and $\rho$: consumers have a lower taste for variety at high $\rho$, which we know from Figure 6, panel A, is when prices increase most. As a result, welfare can fall with globalization. As panel B shows, the gains from globalization are decreasing in both $\varepsilon$ and $\rho$, and are *negative* for sufficiently convex demand: as shown in online Appendix B16, the exact condition for this is $\rho > (\varepsilon^2 + 2\varepsilon - 1)/\varepsilon^2$. This provides, to our knowledge, the first concrete example of the "folk theorem" that globalization in the presence of monopolistic competition can be immiserizing if demand is sufficiently superconvex. Perhaps equally striking is that welfare rises by more for lower values of $\varepsilon$ and $\rho$: estimates based on CES preferences grossly *underestimate* the gains from globalization in much of the subconvex region, just as they fail to predict losses from globalization in the superconvex region.

[37] See Pettengill (1979) and Dixit and Stiglitz (1979). This range has the linear expenditure system ($\rho = 2/\varepsilon$) as its lower bound, and it includes demand functions that are both sub- and superconvex ($\rho \lesseqgtr (\varepsilon + 1)/\varepsilon$).

[38] This extends a result of Vives (1999).

## IV. Empirically Applying the Demand Manifold

So far, we have presented a theoretical framework that highlights the elasticity and convexity of demand as sufficient statistics for a broad class of theoretical results, and shown how they are related to each other via the demand manifold implied by the underlying demand function. Next, we want to illustrate some potential empirical applications of the manifold. We continue to assume, as in previous sections, that the observations come from a monopolistically competitive industry with constant marginal costs. Section IVA shows how, with no further assumptions, an empirical manifold can be estimated if we have information on markups and pass-through coefficients for a sample of firms. Section IVB shows how this empirical manifold can be used to infer income elasticities if we assume in addition that consumer preferences are additively separable. Finally, Section IVC shows how we can go further and use the manifold for welfare analysis if we are willing to make further parametric assumptions about the structure of preferences and the distribution of firm productivities.

### A. *Estimating the Manifold*

Clearly, if we have estimates of a demand function we can directly calculate an empirical manifold. However, this requires taking a stand on the parametric form of demand. An alternative approach is to draw on recently-developed methods that make it possible to proceed without making parametric assumptions about preferences or demand. In particular, the methods developed by De Loecker et al. (2016) for inferring markups and estimating pass-through coefficients do not impose assumptions about the form of demand or the underlying market structure.

Suppose, therefore, that we have a dataset giving information on markups and pass-through coefficients for a sample of firms. In order to apply our approach from previous sections, we need to assume that the observations are generated by a monopolistically competitive industry, with constant marginal costs. Given these assumptions (though without any restrictions on preferences or the distribution of productivities), we can invoke from Section IC the expressions for the proportional markup $m$ and the proportional pass-through coefficient $k$, which we repeat here for convenience:

$$(39)\qquad \text{(i) } m \equiv \frac{p-c}{c} = \frac{1}{\varepsilon-1}; \qquad \text{(ii) } k \equiv \frac{d\log p}{d\log c} = \frac{\varepsilon-1}{\varepsilon}\frac{1}{2-\rho}.$$

Using these, it is straightforward to back out the values of the elasticity and convexity, provided we know (or have estimates of) the markup and the pass-through coefficient:

$$(40)\qquad \text{(i) } \varepsilon = \frac{m+1}{m}; \qquad \text{(ii) } \rho = 2 - \frac{1}{k}\frac{1}{m+1}.$$

This gives a two-dimensional array that can be represented by a "cloud" of points in $(\varepsilon, \rho)$ space.

The next step is to infer from these data an underlying demand manifold. In effect, this means that we want to estimate the parametric manifold that gives the best fit to the data in some sense. This does not mean that we need to assume a particular form of demand itself: as discussed in Section II, we can expect the manifold to be invariant with respect to some demand parameters in most cases. Hence estimating the manifold requires inferring far fewer parameters than would estimating the demand functions themselves.

Actually estimating the demand manifold can be done in different ways. One approach would be to assume a general functional form for the manifold. Such an estimated demand manifold is always numerically integrable, which allows us to infer some of the properties of the implied demand function. Implementing this approach would require firm-level observations on pass-through and markups. We are not aware of any such data available to date. However, we illustrate how the approach can be implemented for the average firm by using the empirical results from De Loecker et al. (2016). They estimate the average pass-through from costs to prices for a sample of Indian firms, and also estimate the markups for each firm. Hence, we can estimate the demand parameters faced by the average firm without making any assumptions about the form of demand.

Alternatively, a more structural approach would be to assume a particular parametric family of demand functions, and to find the manifold from this family that best fits the data. Again, implementing this approach for most demand functions would require firm-level observations on pass-through and markups which we do not have. However, for the CPPT demand function (introduced in Section IIE), which implies the same degree of pass-through but different markups for each firm, we can again illustrate this approach using the data from De Loecker et al. (2016).

In either case, estimating the manifold provides considerable information about the form of demand, and especially about its key implications for comparative statics, without the need to estimate the demand function directly. After briefly discussing the data, we illustrate each of these approaches in turn.

De Loecker et al. (2016) give three estimates of average pass-through, one using ordinary least squares (OLS) and two using instrumental variables (IV). We use the OLS and the first of their IV estimates, for reasons discussed in online Appendix B17. The OLS estimate of $k$ is 0.337 with a 95 percent confidence interval of 0.257 to 0.417; the IV estimate is 0.305 with a 95 percent confidence interval of 0.140 to 0.470. These estimated confidence intervals for $k$ imply estimated confidence intervals for the convexity of demand faced by the average firm, using the expression for constant proportional pass-through from equation (14). These are illustrated by the horizontal boundaries of the shaded regions in Figures 10 and 11: those of the darker region (in Figure 10) correspond to the confidence interval implied by the OLS estimate of the pass-through coefficient, while those of the lighter region (in both figures) correspond to the confidence region implied by the IV estimate.

To implement our first approach, we want to match these estimates of the convexity faced by the average firm with an estimate of the elasticity faced by the same firm. The published data from De Loecker et al. (2016) do not provide this, but do give both the mean and the median of the markup distribution. In the absence of any other information on the markup distribution, we take the range between the mean and the median as a plausible zone of central tendency for the markup of the firm

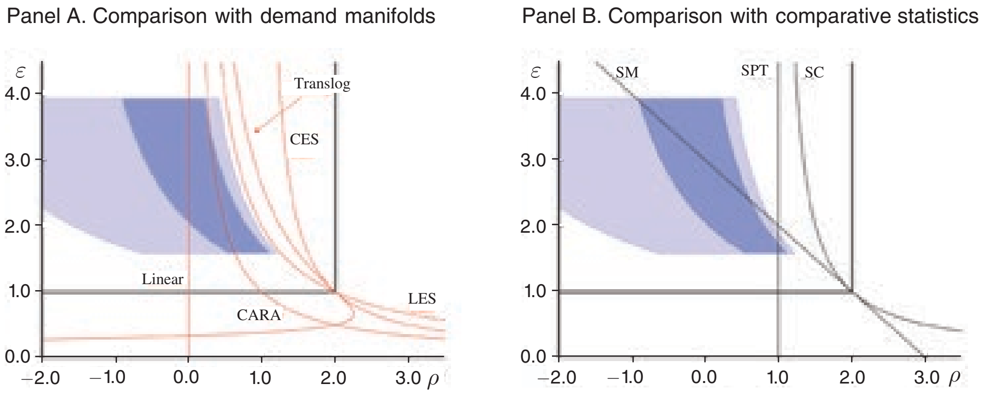


Figure 10. Range of Estimated Values of Elasticity and Convexity of the Average Firm

*Notes:* Shaded areas are confidence regions for $\varepsilon$ and $\rho$ implied by data on $m$ and $k$ from De Loecker et al. (2016). See text for details.

with the average pass-through elasticity. We then use equation (40)(i) to back out estimates of the elasticity faced by this average firm. Combining these estimates with those of the average convexity allows us to construct two "confidence regions" in $(\varepsilon, \rho)$ space for the estimated elasticity and convexity faced by the average firm. Figure 10 illustrates. Panel A shows how the confidence regions compare with the demand manifolds implied by some of the most widely-used demand functions. Clearly, the data are consistent with both linear and CARA demands. By contrast, they are less consistent with LES and translog demands, and even less so with CES. Panel B shows how the confidence regions relate to the regions of comparative statics responses identified in Section I. Given that the results are not consistent with CES demands, it follows that superconvexity is ruled out, which supports the focus in a long literature stemming from Krugman (1979) that concentrates on the "plausible case" of subconvexity. By contrast, neither super- nor submodularity is ruled out: the data are consistent with large firms selecting into activities with either relatively low or relatively high marginal access costs. These conclusions suggest that combining our approach with more detailed estimates of pass-through and markups is likely to prove a fruitful direction for future research.

To implement our second approach, we need to calibrate a particular member of the CPPT class of demand manifolds. This is easily done since it requires only an estimate of the pass-through elasticity $k$. (Recall Figure 2, panel A.) We choose to use the IV estimate of 0.305: the resulting manifold is illustrated by the solid green curves labeled "CPPT" in Figure 11. Panel A shows how the calibrated CPPT manifold compares with the manifolds for some standard demand functions. Panel B yields implications for comparative statics that are similar to those from Figure 10, panel B: superconvexity is clearly ruled out, while supermodularity holds for smaller firms but not for larger ones (those facing an elasticity of 3.28 or lower). A further feature of note is that the manifold implies less than one-for-one absolute pass-through for all but the very largest firms (those facing an elasticity of 1.44 or lower).

Needless to say, these exercises are illustrative only. Nevertheless, the results are very suggestive, and point to further potential applications of our approach, as

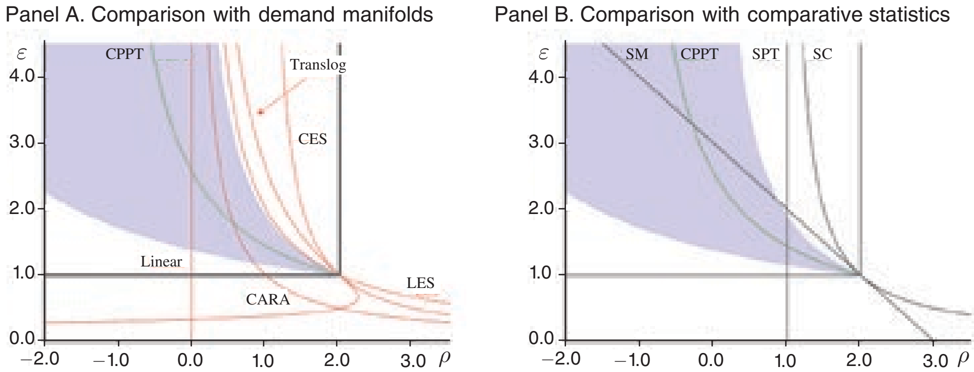


FIGURE 11. CALIBRATED CPPT DEMAND MANIFOLD

*Notes:* Green curves represent the calibrated demand manifold from the CPPT family implied by IV estimates of $m$ and $k$ from De Loecker et al. (2016). Shaded areas give the implied 95 percent confidence interval. See text for details.

datasets with more disaggregated information on pass-through for groups of firms or even individual firms become available.[39]

## B. *Nonparametric Counterfactuals: Income Elasticities*

Additional assumptions are needed to carry out counterfactual analysis. As in Section III, and following a large empirical literature, we assume that preferences are additively separable, though without any parametric restrictions on functional form. This guarantees both manifold invariance (from Corollary 3), and what Deaton (1974) calls "Pigou's Law": income and price elasticities are proportional to each other.[40] In general the relationship is only approximately proportional: see online Appendix B18 for details. However, the error term in the approximation depends on the budget share of the good, so, in the continuum case, the relationship is exactly proportional:[41]

$$\eta_i = \Phi^{-1}\varepsilon_{ii}, \tag{41}$$

where $\eta_i$ and $\varepsilon_{ii}$ are the income and own-price elasticities of good $i$, respectively. The factor of proportionality $\Phi \equiv (d\log\lambda/d\log I)^{-1}$ is the inverse of the elasticity of the marginal utility of income with respect to income. Like $\lambda$ itself, $\Phi$ varies with income and prices; however, it is constant in a cross section, so at a point in time it is the same for all goods. Hence, starting with an empirical manifold estimated from data on pass-through and markups as in Section IVA, we can obtain estimates of the

[39] Estimating pass-through elasticities for individual firms will require panel datasets with sufficient time variation to avoid the need to pool data across firms.

[40] We discuss the case of directly additive preferences only. Similar results hold for indirect additivity, as shown by Deaton (1974).

[41] In empirical applications, it would be natural to allow for deviations from exact proportionality, though in practice they are likely to be very slight if the data are even moderately disaggregated. See equation (B47) in online Appendix B18.

corresponding income elasticities, up to a factor of proportionality.[42] These income elasticities can then be used to carry out counterfactual analyses of prices as done by Simonovska (2015). Note that we do not need to make any assumption about the distribution of firm productivities in this subsection.

Additive separability is of course a restrictive assumption (as discussed by Deaton). However, in empirical applications with many thousands of goods, as is increasingly the norm in applied fields such as international trade, it is essential to impose some structure on the data. Allowing for non-homothetic tastes, even subject to Pigou's Law, seems at least as attractive as imposing homotheticity, as in the widely used CES or homothetic translog specifications. Alternatively, if, as is likely, estimates of income elasticities are available from other sources, this relationship makes it possible in principle to assess the empirical validity of additively separable preferences.

### C. *Parametric Counterfactuals: Gains from Trade*

Further steps are possible if we are willing to make specific parametric assumptions about the form of additively separable preferences and about the distribution of firm productivities. These additional assumptions are the final building blocks needed for counterfactual analyses of welfare, drawing on the results in Section IIIC. The first of these allows us to infer the elasticity of utility of a particular good from the elasticity and convexity of demand for that good. The second allows us to deduce the welfare effects of changes in consumption at the intensive and extensive margins. The demand manifold can be a sufficient statistic for the gains from trade only in the special case of homogeneous firms. However, it is an essential benchmark for, and in some cases gives a lower bound to, the gains from trade in more general models.

## V. Conclusion

In this paper we present a new way of relating the structure of demand functions to the comparative statics properties of monopolistic and monopolistically competitive markets. We begin by illustrating, using a "firm's-eye view" of demand, how the elasticity and convexity of demand determine many comparative statics responses. We then show how the relationship between these two parameters, which we call the "demand manifold," provides a parsimonious representation of an arbitrary demand function, and a sufficient statistic for many comparative statics results. The manifold is particularly useful when it is unaffected by changes in exogenous variables, a property we call "manifold invariance." We characterize the conditions that the elasticity and convexity must exhibit for manifold invariance to hold; we introduce some new families of demand systems that exhibit manifold invariance; and we show that they nest many of the most widely used functions in applied theory. For example,

[42] The levels of the income elasticities in turn can be pinned down using extraneous information, or alternatively, if the goods in question exhaust the consumer's budget, using the Engel aggregation condition $\Sigma_i \omega_i \eta_i = 1$, where $\omega_i$ is the budget share of good $i$.

our "bipower direct" family provides a natural way of nesting translog, CES, and linear demand functions.[43]

To illustrate the usefulness of our approach, we show that the demand manifold allows a parsimonious way of understanding how monopolistically competitive economies adjust to external shocks, as well as a framework for quantifying the effects of globalization. We also show some potential empirical applications of the manifold. The demand manifold can be estimated much more easily than the underlying demand function, using only data on pass-through and markups. With additional assumptions it can in turn be used to calculate income elasticities of demand and the gains from globalization.

Many extensions of our approach naturally suggest themselves. There are many other topics where functional form plays a key role in determining the implications of a given set of assumptions, and where our approach of focusing on the elasticity and convexity of a pivotal function yields important insights. For example, in ongoing work we show that the same approach can be applied to the slope of the demand function, viewed as a function in itself, to derive results on variable pass-through and departures from Gibrat's Law. Further applications to choice under uncertainty and to oligopoly immediately come to mind. As for our application to the effects of globalization in monopolistic competition, the framework we have presented can be extended to allow for trade costs.[44] Finally, the families of demand functions we have introduced provide a natural setting for estimating relatively flexible functional forms, and direct attention toward the parameters that matter for comparative statics predictions.

## Appendix

### A. *Alternative Measures of Slope and Curvature*

As our measure of demand slope, we follow standard practice and work throughout with the price elasticity of demand, which can be expressed in terms of the derivatives of either the inverse or the direct demand functions $p(x)$ and $x(p)$: $\varepsilon \equiv -p/xp' = -px'/x$. An alternative would be to use instead the inverse of this elasticity, $e \equiv -x/px' = 1/\varepsilon$.[45] This has the advantage that, since $e = -xp'/p$, its definition is symmetric with those of demand curvature $\rho \equiv -xp''/p'$, and of $\chi \equiv -xp'''/p''$, a unit-free measure of the third derivative of the demand function, which, following Kimball (1992) and Eeckhoudt, Gollier, and Schneider (1995), we call the "coefficient of relative temperance," or simply "temperance." (This parameter proves useful in some of the proofs and derivations in the online Appendix.) Offsetting advantages of using $\varepsilon$ include its greater intuitive appeal, and the fact

[43] Alternative ways of nesting translog and CES demands, though with considerably more complicated demand manifolds, appear in Novy (2013) and in Pollak, Sickles, and Wales (1984).

[44] The implications of combining trade costs and general non-CES preferences have been considered by Bertoletti and Epifani (2014), Arkolakis et al. (forthcoming), and Mrázová and Neary (2014).

[45] It equals the elasticity of marginal utility: $e = -d\log u'(x)/d\log x$; with additive separability, it has been called the "relative love for variety" by Zhelobodko et al. (2012); and, in monopoly equilibrium, it equals the profit margin or Lerner Index of monopoly power: $e = (p - c)/p$.

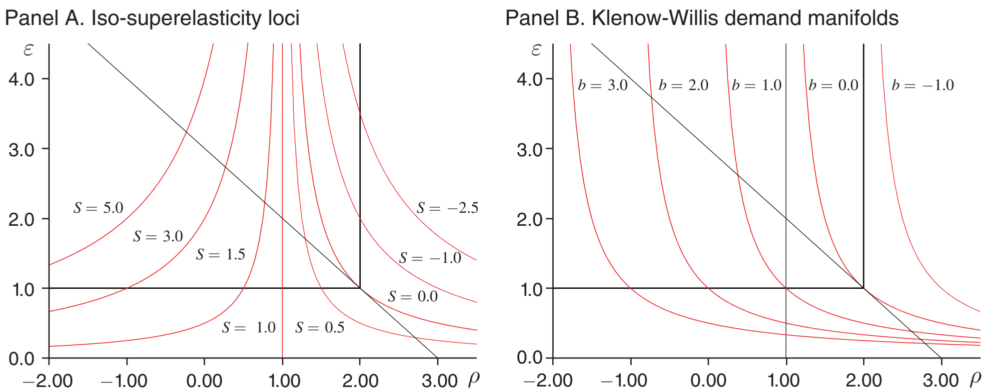


Figure A1. Kimball Superelasticity

that it focuses attention on the region of parameter space where comparative statics results are ambiguous.

Turning to measures of curvature, the convexity of inverse demand which we use throughout equals the elasticity of the slope of inverse demand, $\rho \equiv -xp''/p' = -d\log p'(x)/d\log x$. Its importance for comparative statics results was highlighted by Seade (1980), and it is widely used in industrial organization, for example by Bulow, Geanakoplos, and Klemperer (1985) and Shapiro (1989). An alternative measure is the convexity of the direct demand function $x(p)$: $r(p) \equiv -px''(p)/x'(p)$. A convenient property is that $e$ and $r$ are dual to $\varepsilon$ and $\rho$:

$$e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon}, \qquad r \equiv -\frac{px''}{x'} = \frac{pp''}{(p')^2} = \varepsilon\rho, \tag{A1}$$

$$\varepsilon \equiv -\frac{p}{xp'} = \frac{1}{e}, \qquad \rho \equiv -\frac{xp''}{p'} = \frac{xx''}{(x')^2} = er. \tag{A2}$$

We use these properties in the proof of Proposition 3 in online Appendix B6. Yet another measure of demand curvature, widely used in macroeconomics, is the superelasticity of Kimball (1995), defined as the elasticity with respect to price of the elasticity of demand, $S \equiv d\log\varepsilon/d\log p$. Positive values of $S$ allow for asymmetric price setting in monopolistic competition. It is related to our measures as follows: $S = (d\log\varepsilon/d\log x)(d\log x/d\log p) = (x\varepsilon_x/\varepsilon)(-\varepsilon) = \varepsilon + 1 - \varepsilon\rho$ (the last step using online Appendix B1), so it is positive if and only if demand is subconvex. Figure A1, panel A, illustrates loci of constant superelasticity, $\rho = (\varepsilon + 1 - S)/\varepsilon$. Formally, they correspond to the family of Pollak manifolds, $\bar{\rho}(\varepsilon) = (\sigma + 1)/\varepsilon$, displaced rightwards to be symmetric around the log-linear ($\rho = 1$) rather than the linear ($\rho = 0$) demand manifold.

Kimball himself did not present a parametric family of demand functions. Klenow and Willis (2016) introduce a parametric family which has the property that the superelasticity is a linear function of the elasticity: $S = b\varepsilon$. Substituting for $S$ leads to the family of demand manifolds $\bar{\rho}(\varepsilon) = ((1-b)\varepsilon + 1)/\varepsilon$, which are lateral displacements of the CES locus. Figure A1, panel B, illustrates some members of this family.

We note in footnotes some implications of these alternative measures. The choice between them is largely a matter of convenience. We express all our results in terms of $\varepsilon$ and $\rho$, partly because this is standard in industrial organization, partly because (unlike $e$ and $r$) the inverse demand functions are easily integrated to obtain the direct utility function, and partly because (unlike $\varepsilon$ and $S$) they lead to simple restrictions on the shape of the demand manifold as shown in Proposition 3. However, our results could also be expressed in terms of $e$ and $r$ or of $\varepsilon$ and $S$.

### B. *Oligopoly*

We consider only monopoly and monopolistic competition in the text, but our approach can also be applied to oligopolistic markets. Even in the simplest case of Cournot competition between $n$ firms producing an identical good, this leads to extra complications. Now we need to distinguish market demand $X$ from the sales of an individual firm $i$, $x_i$, with the elasticity and convexity of the demand function $p(X)$ defined in terms of the former: $\varepsilon \equiv -p/Xp'$ and $\rho \equiv -Xp''/p'$. The first-order condition is now $p + x_i p' = c_i \geq 0$, while the second-order condition is $2p' + x_i p'' < 0$. The implied restrictions on the size of the admissible region can be expressed in terms of market shares ($\omega_i \equiv x_i/X$). The first-order condition implies that $\varepsilon \geq \max_i(\omega_i)$, which attains its lower bound of $1/n$ when firms are identical. As for the second-order condition, it implies that $\rho < 2\min_i(1/\omega_i)$, which attains its upper bound of $2n$ when firms are identical. Since (except when firms are identical) market shares are endogenous, the same is true of the size of the admissible region. A different restriction on convexity comes from the stability condition: $\rho < n+1$. This imposes a tighter bound than the second-order condition provided the largest firm is not "too" large: $\max_i(\omega_i) < 2/(n+1)$. Relative to the monopoly case, the admissible region expands unambiguously, except in the boundary case of a dominant firm, where $\max_i(\omega_i) = 1$.

Equally important in oligopoly, as we know from Bulow, Geanakoplos, and Klemperer (1985), is that many comparative statics results hinge on strategic substitutability: the marginal revenue of firm $i$ is decreasing in the output of every other firm. This is equivalent to $p' + x_i p'' < 0, \forall i$, which in our notation implies a restriction on convexity that lies within the admissible region: $\rho < \min_i(1/\omega_i) \geq 1$, which attains its upper bound of $n$ when firms are identical.

### C. *Examples Illustrating Proposition 2: Manifold Invariance*

Sections IIC and IID give details of demand functions that satisfy the conditions of Proposition 2 for manifold invariance to hold. Here we discuss two very different classes of demand function that also exhibit manifold invariance.[46]

[46] If we assume additive separability, both of these demand functions imply hypergeometric sub-utility functions, which can be used as a basis for quantitative studies of welfare questions.

The first of these is the CPPT or constant proportional of pass-through family, discussed in Section IIE. The implied expressions for the elasticity and convexity as functions of output $x$, are

$$\varepsilon(x) = \frac{x^{\frac{k-1}{k}} + \gamma}{\gamma}, \qquad \rho(x) = 2 - \frac{1}{k}\frac{x^{\frac{k-1}{k}}}{x^{\frac{k-1}{k}} + \gamma}. \tag{A3}$$

The requirements that prices be positive, that $\varepsilon$ be greater than one, and that $\rho$ be less than two, imply that all three coefficients, $\beta$, $\gamma$, and $k$, must be positive. The coefficient $k$ also determines whether the demand function is sub- or superconvex: CPPT demands are subconvex, with $\varepsilon_x \leq 0$, if and only if $k$ is less than one. It follows immediately from (A3) that $\varepsilon(x)$ and $\rho(x)$ satisfy condition (11a) from Proposition 2 for manifold invariance with respect to $\gamma$: they depend on $\gamma$ only through a common factor $f = F(x,\gamma) \equiv \gamma^{-1}x^{(k-1)/k}$. Thus $\varepsilon = f + 1$ and $\rho = 2 - (f+1)/fk$; combining recovers the manifold as in (6). Note that, although all firms have the same elasticity of pass-through, this demand function allows for variable markups. Writing the markup as $m \equiv (p - c)/c = 1/(\varepsilon - 1)$:

$$m(x) = \frac{\gamma}{x^{\frac{k-1}{k}}}. \tag{A4}$$

Naturally, markups are increasing in firm size, $m_x \geq 0$, if and only if demands are subconvex, $k \leq 1$.

The second class of demand function that exhibits manifold invariance is a generalization of the "CREMR" demand function introduced in Mrázová, Neary, and Parenti (2015):

$$p(x) = \beta x^{-\eta}(x - \gamma)^{-\theta}. \tag{A5}$$

When $\eta = 1$, this reduces to the CREMR case, so-called because it exhibits a constant revenue elasticity of marginal revenue. For the demand function in (A5) the elasticity and convexity can be written as

$$\varepsilon = (\eta + \theta f)^{-1} \quad \text{and} \quad \rho = \eta + \theta f + \frac{\eta + \theta f^2}{\eta + \theta f}, \tag{A6}$$

where $f = F(x,\gamma) \equiv x/(x - \gamma)$. These clearly satisfy condition (11a) from Proposition 2: both elasticity and convexity depend on $x$ and $\gamma$ through a common function $F(x,\gamma)$. Hence the demand manifold is invariant with respect to the parameter $\gamma$:

$$\bar{\rho}(\varepsilon) = \frac{1}{\theta\varepsilon}\left[(\eta + \theta)\eta\varepsilon^2 - 2\eta\varepsilon + \theta + 1\right]. \tag{A7}$$

### D. *Heterogeneous Firms with Additive Separability*

The first step is to calculate the elasticities of the maximum operating profit function:

$$\pi(c,\lambda,k) \equiv \max_y\,[p(y,\lambda,k) - c]y \quad \text{where} \quad p(y,\lambda,k) = \lambda^{-1}u'(y/kL). \tag{A8}$$

For later use, the first and second derivatives of the inverse demand function, expressed in terms of elasticities, are

$$\text{(A9)} \qquad \frac{yp_y}{p} = -\frac{1}{\varepsilon}, \quad \frac{\lambda p_\lambda}{p} = -1, \quad \text{and} \quad \frac{kp_k}{p} = \frac{1}{\varepsilon};$$

$$\text{(A10)} \qquad \frac{yp_{yy}}{p_y} = -\rho, \quad \frac{yp_{y\lambda}}{p_\lambda} = -\frac{1}{\varepsilon}, \quad \text{and} \quad \frac{yp_{yk}}{p_k} = \frac{1}{\varepsilon}.$$

Using the envelope theorem, the derivatives of the profit function are:

$$\text{(A11)} \quad \pi_c = -y, \quad \pi_\lambda = -\lambda^{-2}u'y = -\lambda^{-1}py, \quad \pi_k = -\frac{y^2u''}{\lambda k^2 L} = -\frac{y^2}{k}p_y.$$

These in turn can be expressed in terms of elasticities, making use of the first-order condition $p + yp_y = c$:

$$\text{(A12)} \quad \frac{c\pi_c}{\pi} = -\frac{cy}{\pi} = -\frac{c}{p-c} = -(\varepsilon - 1), \quad \frac{\lambda\pi_\lambda}{\pi} = -\frac{py}{\pi} = -\frac{p}{p-c} = -\varepsilon,$$

$$\text{(A13)} \quad \frac{k\pi_k}{\pi} = -\frac{y^2p_y}{(p-c)y} = \frac{yp_y}{yp_y} = 1.$$

Aggregating these gives the elasticities of aggregate profits:

$$\text{(A14)} \quad \frac{\lambda\bar{v}_\lambda}{\bar{v}} = \int_{\underline{c}}^{\bar{c}} \frac{v(c,\lambda,k)}{\bar{v}(\lambda,k)} \frac{\lambda\pi_\lambda(c)}{\pi(c)} g(c)\,dc = -\int_{\underline{c}}^{\bar{c}} \frac{v(c,\lambda,k)}{\bar{v}(\lambda,k)} \varepsilon(c)g(c)\,dc = -\bar{\varepsilon},$$

$$\text{(A15)} \quad \frac{k\bar{v}_k}{\bar{v}} = \int_{\underline{c}}^{\bar{c}} \frac{v(c,\lambda,k)}{\bar{v}(\lambda,k)} \frac{k\pi_k(c)}{\pi(c)} g(c)\,dc = \int_{\underline{c}}^{\bar{c}} \frac{v(c,\lambda,k)}{\bar{v}(\lambda,k)} g(c)\,dc = 1,$$

where the final step makes use of the definition of $\bar{v}(\lambda, k)$ from (28). Note that these aggregate effects are a weighted average of the elasticities of *operating profits*, weighted by each firm's share in *expected total profits*; naturally, the weights attached to firms that choose not to enter and so earn zero profits are themselves zero.

Using these results, we can solve for the effect of globalization on the degree of competition $\lambda$ by totally differentiating the zero-expected-profit condition (28):

$$\text{(A16)} \qquad \hat{\lambda} = -\left(\frac{\lambda\bar{v}_\lambda}{\bar{v}}\right)^{-1} \frac{k\bar{v}_k}{\bar{v}} = \frac{1}{\bar{\varepsilon}}\hat{k}.$$

The next step is to solve for the effects of globalization at the intensive margin. Totally differentiating the first-order condition, and making use of (A9) and (A10), the partial elasticities of output with respect to $\lambda$ and $k$ are given by

$$\text{(A17)} \qquad \frac{\lambda y_\lambda}{y} = -\frac{\varepsilon - 1}{2 - \rho} \quad \text{and} \quad \frac{ky_k}{y} = 1.$$

Hence, the total derivative of output with respect to $k$, allowing for the indirect effect via the level of competition, is

$$\hat{y} = \left(\frac{ky_k}{y} + \frac{\lambda y_\lambda}{y}\frac{k}{\lambda}\frac{d\lambda}{dk}\right)\hat{k} = \left(1 - \frac{\varepsilon - 1}{2 - \rho}\frac{1}{\bar{\varepsilon}}\right)\hat{k}, \tag{A18}$$

where we use (A16) and (A17) to simplify. Rearranging gives the decomposition in (31) in the text.

As for prices, equation (20) in Section IIIA, which relates price changes to changes in per capita consumption $x$, continues to hold for each individual firm. The change in $x$ in turn can be derived from the goods-market equilibrium condition (22):

$$\hat{x} = \hat{y} - \hat{k} = \left(1 - \frac{\varepsilon - 1}{2 - \rho}\frac{1}{\bar{\varepsilon}} - 1\right)\hat{k} = -\frac{\varepsilon - 1}{2 - \rho}\frac{1}{\bar{\varepsilon}}\hat{k}. \tag{A19}$$

Substituting into (20) and rearranging gives the change in prices in (32).

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